Private Overborrowing under Sovereign Risk *

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Abstract

This paper proposes a quantitative theory of the interaction between private and public debt in an open economy. Excessive private debt increases the frequency of financial crises. During such crises, the government provides fiscal bailouts financed with risky public debt. This response may cause a sovereign debt crisis, which is characterized by a higher probability of a sovereign default. The model is quantitatively consistent with the evolution of private debt, public debt, and sovereign spreads in Spain from 1999 to 2015, and provides an estimate of the degree of overborrowing, its effect on sovereign risk, and optimal macroprudential policy.

Keywords: Bailouts, credit frictions, financial crises, macroprudential policy, sovereign default

JEL Classifications: E32, E44, F41, G01, G28

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1 Introduction

A feature of the 2010–2015 European Debt Crisis is that some of the countries facing the most significant spikes in borrowing costs had previously maintained fiscally conservative policies. Spain stood out as a prime example of this stance. In fact, from the inception of the Euro in 1999 until the global financial crisis in 2008, Spain consistently adhered to the budgetary and public debt limits outlined by the Stability and Growth Pact. During this same period, however, Spain amassed a large stock of international private debt, primarily in its banking sector.[1] As financial turmoil intensified, the government implemented numerous bailouts for heavily indebted financial institutions. These actions resulted in a sudden surge in Spain's public debt and its interest rate spreads. Consequently, questions have emerged regarding the relationship between private debt crises and sovereign risk, as well as how governments facing default risk should address systemic vulnerabilities in international private credit markets.[2] This paper is among the first few to provide a joint analysis of the interplay of private debt and sovereign risk, which is necessary to provide adequate policy prescriptions.

This paper offers quantitative insights into three key questions. First, was the Spanish private sector excessively indebted in the lead-up to the crisis, and, if so, by how much? Second, what was the effect of excessive private debt on the severity of the sovereign debt crisis that followed? Third, how do the optimal macroprudential policy prescriptions change when one takes sovereign risk into account?

To address these questions, we construct a small open economy model that incorporates both financial crises stemming from collateral constraints on private debt and sovereign default crises resulting from long-term defaultable public debt. Firstly, the model aligns quantitatively with Spanish data, providing an estimate of excessive private debt stock at an average of 8% of gross domestic product (GDP). Secondly, the model captures the dynamics of private debt, public debt, and sovereign spread during the 2012 crisis, revealing that excessive sovereign risk elevated default risk by 0.7 percentage points (p.p.). Thirdly, considering sovereign risk, the optimal macroprudential tax would increase by an average of 0.4 p.p., with its volatility rising by 1.0 p.p.

Private debt is modeled as in Mendoza (2002) and Bianchi (2011), and the sovereign debt structure follows the tradition of Eaton and Gersovitz (1981) and Arellano (2008) with long-term bonds as in Hatchondo and Martinez (2009). We solve two versions of the model. In the baseline version, a con-

[1] Morris et al. (2006) discusses the 2005 reform of the Stability and Growth Pact, praising Spain's adherence to its regulations.Schuknecht et al. (2011) document Spanish compliance with deficit and sovereign debt regulations until the 2008 recession. Lane (2013),Chen et al. (2013), Hale and Obstfeld (2016) and Hobza and Zeugner (2014) discuss the current account imbalances of periphery European countries and document a flow in the form of debt instruments from "core" countries toward financial institutions in the periphery. In't Veld et al. (2014) and Ratto and Roegera (2015) link the increase in capital flows to Spanish banks financing a boom in the construction sector.

[2] Private credit booms have been linked to subsequent sovereign debt crises before. An earlier literature analyzing the 1997 currency crises in Thailand, Korea, and Indonesia stresses this link. Burnside et al. (2001) argue that implicit bailout guarantees lead to private credit booms and raise expectations of large fiscal deficits in the future. Schneider and Tornell (2004) show that systemic bailout guarantees cause both credit cycles and self-fulfilling crises.

tinuum of identical households makes the private borrowing decisions and a benevolent government makes the taxes, default, and public borrowing decisions. In the normative version, a benevolent social planner (SP) makes aggregate borrowing decisions about both private and public debt and then transfers the proceeds to the households that make all consumption choices. Thus, the planner and the competitive households are subject to the same market clearing conditions, as well as credit constraints. Nevertheless, the planner's choice of allocations may be different from that of the competitive equilibrium because the planner internalizes the general equilibrium effects of the aggregate choices that are made. We show that the planner's allocations can be decentralized by extending the baseline framework to allow the government to impose state-dependent taxes on private borrowing. We also find that the socially planned version features a lower level of private debt, a lower level of public debt, and a lower interest rate spread. These differences allow the planner to achieve a higher level of welfare.

In the quantitative section, we first calibrate the baseline version of the model to the Spanish data from 1999 to 2012. This calibration ensures alignment with the pre-crisis Spanish economic landscape characterized by low public debt, high private debt, and nearly negligible interest rate spreads. Subsequently, we utilize the calibrated parameters to solve the socially planned version of the model. A comparison between the socially planned economy and the baseline model, focusing on their respective ergodic distributions, serves as the basis for quantifying the excess private debt stock and its impact on sovereign risk.

We then use the 2008-2015 Spanish data to simulate the crisis in the model. We feed the observed exogenous shocks from the data and infer the unobserved shocks using the particle filter approach proposed by Bocola and Dovis (2019). As in the data, the baseline model's government opts for substantial bailouts to the private sector, financed through external public debt, in response to the 2012 shocks. This response coincides with a large deleveraging in private debt and a rise in the public interest rate spread commensurate with the increase observed in Spain. Conversely, facing identical shocks, the social planner maintains the interest rate spread on public debt below 1% and achieves a more gradual reduction in private debt.

Lastly, we compute the optimal macroprudential policies necessary to implement the planner's allocations. We compare these policies with those required for efficiency in an economy devoid of public debt and sovereign risk. Our analysis reveals that in the presence of sovereign risk, macroprudential policies need to be more stringent. Specifically, compared to the Bianchi (2011) economy, optimal macroprudential policies exhibit increases in their first and second moments.

The fundamental insight of the paper is understating why the baseline and socially planned allocations differ. The two sources of private overborrowing are general equilibrium effects which are accounted for by the social planner but not by the competitive households in the baseline version.

The first source of overborrowing arises from the fact that the value of collateral available to households depends on market-determined prices. Mendoza (2002) named this effect Fisherian debt-deflation. Unlike households, the planner internalizes the effect of debt issuances on future non-tradable prices and borrowing capacity. Consequently, the baseline version exhibits a higher level of private debt, more frequent periods when the credit constraint binds, and sharper contractions in private debt and consumption during these periods.

The second source of overborrowing stems from the fact the stock of private debt changes future government policies and hence the price of public debt. This source combines the insights from the sovereign default literature with multiple assets, Arellano and Ramanarayanan (2012) and Hatchondo et al. (2016), where one type of asset affects the default risk of the other, with those on the literature with decentralized borrowing and centralized default, Kim and Zhang (2012) and Jeske (2006).

In the model, these two sources of overborrowing interact in nontrivial ways in the lead-up to and during financial crises. When the credit constraint on private debt binds, a reduction in consumption decreases the value of collateral, necessitating further deleveraging in the private sector and thereby causing an even greater decline in consumption. To break this negative feedback loop, the government resorts to publicly financed bailouts. However, this response comes at the cost of higher sovereign risk. In anticipation of more frequent bailouts in a decentralized economy, lenders demand a higher spread, thereby hampering the government's ability to respond to crises.[3]

Related Literature: The paper belongs to quantitative sovereign default literature introduced in Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008) to explain the business cycles in emerging economies.

The model incorporates a long-term structure for public debt taken from Hatchondo and Martinez (2009) while keeping, for simplicity's sake, the short-term maturity in private debt. As explained in Chatterjee and Eyigungor (2012) long-term debt public debt generates dynamics of the interest rate spread that are more consistent with the data. Moreover, by focusing on an environment with two assets of different maturities it is closely related to Arellano and Ramanarayanan (2012) and Hatchondo et al. (2016). Consistent with these papers, we find that long-term debt serves as a hedge against rollover risk and can substitute short-term debt. However, we depart from this literature by assuming that the private sector controls the issuance of short-term debt and the government can't default on this asset on behalf of the private sector. Thus private debt carries no endogenous default risk in our framework.

The paper is closely related to the branch of the sovereign debt literature that studies the links between sovereign debt and the private economy. In contrast, to Mendoza and Yue (2009) the analysis presented here assumes that private agents retain access to international credit markets even during sovereign default episodes. In complementary work, Arellano et al. (2019) study a sovereign debt

[3]Note that expectations of future bailouts in case of adverse shocks are not the cause of private overborrowing here. Bailouts in this model depend on the aggregate state of the economy and the exogenous shocks; thus individual households do not expect their borrowing choices to affect government policies. See Bianchi (2016) for an environment where this is not the case.

model with endogenous production with firms and banks subject to credit frictions, but where private credit is domestic. We assume instead an endowment economy, but we allow for a private sector with access to international credit markets. We believe that this assumption is better suited for Spain where private external debt was large, while the country of interest in Arellano et al. (2019) is Italy, where most debt was domestically held.[4]

The paper is also related to the literature that studies the trade-offs between centralized and decentralized borrowing. With complete markets, Jeske (2006) and Wright (2006) find that a centralized environment, where only the government can issue international debt and default on it, is preferable to a decentralized environment where individual households make the borrowing and default choices. With incomplete markets, Kim and Zhang (2012) analyzes an environment where decentralized households make the borrowing choices and a centralized government makes the default choice for all agents. Their economy can feature both over or underborrowing depending on the default costs. Our paper incorporates multiple assets to the incomplete markets framework, overborrowing is driven primarily by private assets.

Furthermore, the paper contributes to the literature on credit frictions, financial crises, and macroprudential policies. In particular, it belongs to the branch on systemic credit risk (see Lorenzoni (2008), Bianchi (2011), and Dávila and Korinek (2018)) and its management with taxes on private borrowing (see Bianchi and Mendoza (2018), Farhi and Werning (2016), and Jeanne and Korinek (2019)). The paper also shows that government bailouts financed with external defaultable debt are not a substitute for optimal macroprudential policies. The role of bailouts in the model is similar to the one found in Bianchi (2016), Keister (2016), and Chari and Kehoe (2016). In contrast to those papers, we distinctly assume here that the bailouts are paid for with long-term strategically defaultable debt. This feature allows the model to create a path from financial crises to sovereign debt crises– a relationship observed in the data.[5]

By analyzing how private credit affects the sovereign spread, we also contribute to a growing literature on the feedback loop between sovereigns and the domestic financial sector referred to as "doom loops." Theoretical models of this issue are presented in Korinek (2012), Brunnermeier et al. (2016), and Farhi and Tirole (2018). A subset of this literature, exemplified by Perez (2015), Bocola (2016), and Sosa-Padilla (2018), has focused on developing quantitative models that capture only a part of this loop, the transmission of sovereign risk to private risk. This paper complements the existing quantitative literature by focusing on the other part of the loop—where a financial crisis in the private sector precipitates a sovereign debt crisis. In the model, excessive private credit will

[4]A recent related paper that also allows for international private debt is Kaas et al. (2020). The main difference is that private debt in our model is inefficiently high from a social perspective, and this inefficiency increases the incidence and magnitude of crises. As a result, the frequency of public bailouts, in response to reductions in the borrowing capacity in the private sector, is an endogenous outcome of the model.

[5]The literature on bailouts also deals extensively with the issue of moral hazard that the expectation of government bailouts induces. This concern is not addressed in this paper because households take as given that government policies are functions of aggregate states and not their individual actions.

endogenously generate financial crises and increase the incentives for government interventions that increase default risk and spreads.

2 Motivation: The path of debt and spreads in Spain, 1999-2015

This section documents the evolution of international private and public debt in Spain from the creation of the Eurozone in 1999 to the end of the Spanish sovereign debt crisis in 2015. We show that an initial phase characterized by a large accumulation of private debt, alongside relatively low levels of public debt and minimal spreads, gave way to a sudden reduction in private debt and the onset of a sovereign debt crisis. Although our primary focus is on Spain, Reinhart and Rogoff (2011), Lane (2013), and Gennaioli et al. (2018) have recorded similar dynamics in other countries and periods.

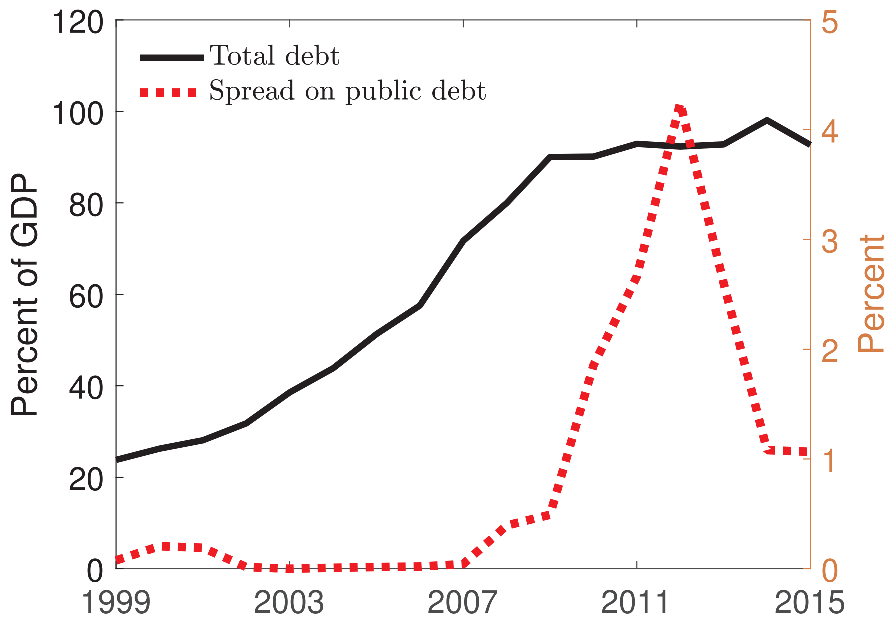

Figure 1: Total international debt and sovereign spread

Note: Total debt corresponds to the inverse of the international investment positions. The spreads correspond to the average difference between the interest rate on a Spanish six-year treasury bill and the interest rate on the German equivalent. The data source for debt is the Bank of Spain, and the interest rate data are from Bloomberg. All debt data is annualized. Total debt aggregates of both the private and public positions. The public position comprises assets held by the public administration and the Bank of Spain, while the private position encompasses all other assets. Appendix C provides more details on the sources and calculations of the debt positions. The average maturity of public debt in Spain during this period was six years. Section 4 explains how the average maturity is computed.

Figure 1 shows the evolution of total debt accumulation and spreads during this period. On the left axis, we depict the evolution of the international investment position as a percentage of GDP presented on an inverted scale where positive values denote net liabilities. While this measure encompasses all types of assets, we refer to net international liabilities as debt throughout the paper. On the right axis, we show the spread defined as the difference between the interest rate paid on a Spanish six-year treasury bond and its German counterpart. The figure shows an initial period of debt accumulation between 1999 and 2008, followed by a period where total debt remained constant at around 92% of GDP. Interest rate spreads remained near zero until 2009, after which they grew slowly and then surged in 2012. Some observers, such as Banco de España (2017), have found it challenging to reconcile rational financial market behavior with a phase of rapid debt escalation but minimal spreads (1999–2008), followed by a period of surging spreads despite steady total debt (2009–2015). This paper provides a theory that explains both periods.

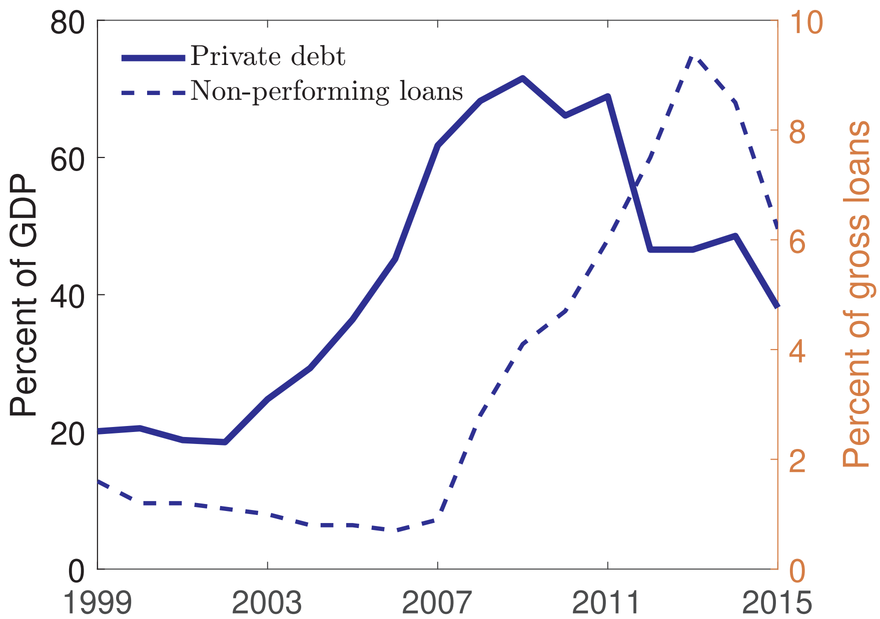

Figure 2: Private debt and nonperforming loans

Note: Private debt corresponds to the inverse of the international investment positions of the financial and nonfinancial private sector. Nonperforming loans are computed as a share of total gross loans. The data source for debt is the Bank of Spain, and the loans data are from Bloomberg. More details can be found in Appendix C.

Figure 2 shows private debt and domestic nonperforming loans over the same time period. The left axis represents the private sector's debt position as a percentage of GDP (solid line), while the right axis represents nonperforming loans as a percentage of gross loans (dashed line). Net liabilities in the private sector surged from 20% of GDP in 1999 to 70% of GDP in 2009. Following a slight decline over two years, private debt experienced a sudden 22% drop in GDP in 2012.

As noted by International Monetary Fund (2012), International Monetary Fund (2014), and Martin et al. (2019), among others, the accumulation of external private debt was largely driven by a banking sector financing a construction boom. However, as housing prices decreased and mortgages went unpaid, rolling over private debt abroad became increasingly difficult. Figure 2 shows that the increase in private debt ceased around the same time as the rise in nonperforming loan shares. Furthermore, the abrupt drop in 2012 coincided with the high mark of the share of defaults. On average, 7.5% of gross loans were nonperforming between 2011 and 2015. Consequently, the model we propose will incorporate a domestic financial shock calibrated to the evolution of nonperforming loans.

Finally, Figure 3 completes the analysis by showing the joint evolution of public and private debt. Combined, these two series add up to the total debt presented in Figure 1. The symmetry between these two aggregates highlights the importance of the debt decomposition presented here. Public

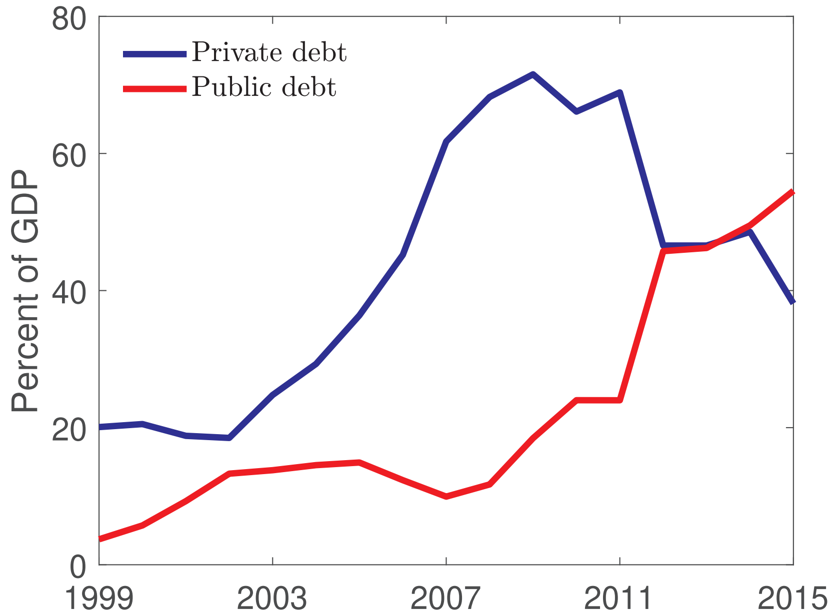

Figure 3: Private and public debt

Note: Private debt corresponds to the inverse of the international investment positions of the financial, and nonfinancial private sector. Public debt corresponds to the inverse of the international investment position of the Bank of Spain and other public administrations. The data source is the Bank of Spain. More details can be found in Appendix C.

external debt in Spain remained below 20% of GDP from 1999 to 2007. In constrast, from 2008 to 2015, public external debt escalated from 11% to 55% of GDP. Notably, the most substantial yearly increase occurred in 2012, with public liabilities rising by 22% of GDP, precisely mirroring the drop in private debt. As observed in Banco de España (2017), this symmetry is not coincidental. Between 2008 and 2012, the Spanish government provided financial assistance to its lending institutions primarily through bailouts and transfers of toxic assets. Direct aid to the Spanish banking sector totaled 70 € billion, equivalent to around 7% of GDP, with a majority of these funds transferred by the newly established Fund for the Orderly Restructuring of the Banking Sector.[6]

In summary, the pre-crisis period spanning 1999 to 2007 saw substantial accumulations of private debt alongside low public debt and minimal public spreads. This phase was succeeded by a sluggish recovery period, spanning 2008 to 2011 in the data. During these years, there was a rise in nonperforming loans in the private sector alongside moderate private deleveraging. Concurrently, public debt and spreads increased but remained relatively subdued. The subsequent period, from 2012 to 2015, marked the onset of the financial and sovereign debt crisis. These years were characterized by significant public bailouts aimed at reducing net liabilities in the private sector, which were financed

[6]Beyond direct transfers, private debt declined following liquidation of private assets while public debt increased to finance unemployment benefits and economic stimulus programs to mitigate the financial crisis. A full overview of the restructuring of the Spanish financial sector is beyond the scope of this paper. More details can be found in International Monetary Fund (2010) and Banco de España (2017).

through the issuance of public debt. The symmetric evolution of debt positions coincided with notable increases in the interest rate spread on public debt. The following proposes a theoretical framework generating dynamics consistent with these empirical observations.

3 A model of financial and sovereign debt crises

This section presents a dynamic small open-economy model with one-period international private bonds subject to an occasionally binding borrowing constraint, as in Bianchi (2011), and long-term, strategically defaultable international public bonds, as in Hatchondo and Martinez (2009).

3.1 Environment

Time is discrete and indexed by $t \in \{0, 1, ..., \infty\}$. The economy consists of a continuum of identical households of unit measure, a benevolent government, and a continuum of risk-neutral, competitive foreign creditors who lend to both domestic agents via two distinct assets. The emphasis is on real values rather than nominal ones due to the predominance of Spanish debt denominated in euros, whose supply is regulated by the European Central Bank.[7]

Households preferences: The representative household has an infinite life horizon and preferences given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

where $\mathbb{E}_0$ is the expectation operator conditional on date 0 information; $0 < \beta < 1$ is a discount factor; and $u(\cdot)$ is a standard increasing, concave, and twice continuously differentiable function satisfying the Inada condition. The consumption basket c is an Armington-type constant elasticity of substitution (CES) aggregator with an elasticity of substitution $1/(\eta + 1)$ between tradable goods c^T and nontradable goods c^N, given by

$$c = \left[\omega \left(c^T\right)^{-\eta} + (1-\omega)\left(c^N\right)^{-\eta}\right]^{-\frac{1}{\eta}}, \eta > -1, \omega \in (0, 1).$$

Income shock: Each period the economy receives a stochastic endowment of tradable goods $y^T \in \mathbb{R}^+$ and a constant endowment of nontradable goods $y^N \in \mathbb{R}^+$. The numeraire is the tradable good. Tradable endowments are drawn from a first-order Markov process independently of all other shocks. Following Bianchi et al. (2016) and Rojas and Saffie (2022), we assume a constant endowment of nontradables equal to one. This assumption suffices to induce fluctuations in aggregate output consistent

[7] The interaction of sovereign default and the inability to inflate away debt within the context of the European Debt Crisis are examined in Aguiar et al. (2014) and Aguiar et al. (2015). For the specific case of Spain, Bianchi and Mondragon (2018) investigate this issue within a framework featuring nominal rigidities.

with empirical data (Appendix G) and matches recession patterns observed in our validation exercise (Section 6). We relax this assumption in Appendix I and find consistent results.

Private debt and financial shocks: Households have access to one-period non-state-contingent debt denominated in units of tradables. Following the standard convention, b denotes the individual level of private debt and B denotes the aggregate level. Each period a stochastic fraction π of these bonds is defaulted on. Incorporating these private default shocks enables the model to capture the dynamics of domestic credit in Spain, referred to as the domestic financial shock. Similar to income shocks, the domestic financial shock is drawn from a first-order Markov process independently of all other stochastic shocks. Additionally, private bond issuances are subject to a collateral credit constraint, outlined as follows:

$$q_t b_{t+1} \leq \kappa_t \left(y_t^T + p_t^N y^N \right), \tag{2}$$

where q_t is the price of private bonds, and p_t^N is the equilibrium price of nontradable goods in units of tradables. The market value of private debt issuances $q_t b_{t+1}$ is capped at a fraction $\kappa_t \geq 0$ of current income. This credit constraint succinctly captures the empirical observation that income plays a critical role in determining access to credit markets. Theoretically, the constraint can be derived as an incentive-compatibility constraint on borrowers when limited enforcement prevents lenders from collecting more than a fraction κ_t of the value of the endowment owned by a defaulting household. Nontradable goods are included in the collateral constraint because, although foreign creditors may not value them, we assume they can be seized in the event of default and sold in exchange for tradable goods in the domestic market. Given the common use of collateral constraints in mortgage lending, this assumption is particularly relevant in the Spanish context, where mortgage loans played a significant role in the expansion of private credit.[8]

The international financial shock κ_t, is the fraction of market income required as collateral and is drawn from a first-order Markov process. Stochastic changes in collateral requirements can be interpreted as shocks to creditors' risk assessments of borrowers. Such financial shocks have been demonstrated to be capable of explaining the dynamics of private financial crises in advanced economies (Boz and Mendoza (2014)) as well as balance of payment crises in emerging economies (Mendoza (2002)). From a modeling perspective, these shocks generate fluctuations in private borrowing that are independent of fluctuations in other domestic fundamentals. This feature is in line with empirical

[8]The current, rather than the future, price appears in the constraint because the opportunity to default occurs at the end of the current period, before the realization of future shocks. See Bianchi and Mendoza (2018) for a derivation of a similar constraint. Note that while private debt is explicitly modeled here as issued internationally by households, the same constraint arises under a broader set of assumptions. Specifically, we could assume that credit is provided to households by a competitive domestic financial system with unrestricted access to global capital markets but subject to the same enforcement friction. As discussed in Section 2, this interpretation aligns more closely with the events in Spain, where commercial and savings banks borrowed internationally and then allocated these funds to households and construction firms. Lastly, the assumption of short-term maturity is consistent with empirical literature, Gorton et al. (2020) and Chen et al. (2019), documenting a decrease in the maturity of private bonds issued in advanced economies during this period.

findings by Forbes and Warnock (2020), who observe that shocks in international volatility, monetary policy, or sudden-stop crises in similar or neighboring countries influence lenders' perceptions of the private sector's solvency. In particular, these shocks enable the model to accommodate changes in investors' behavior toward Eurozone banks following the Greek financial crisis.

The domestic financial shock, π_t, corresponds to the exogenous share of private bonds defaulted on each period and will in equilibrium determine the value of q_t. We include this shock to capture the domestic financial frictions that affected Spain at the peak of the 2012 crisis. In the lead-up to the crisis, the balance sheet of Spanish banks deteriorated significantly. The share of gross nonperforming loans steadily increased between the end of the great financial crisis and 2013. From a modeling perspective, this shock guarantees that not all private debt will be fully repaid but also that private debt will be issued at a spread relative to the risk-free rate. Thus, the economy's preference for private debt over public debt on average will not be driven by a constant advantage in terms of interest rate.

We incorporate both domestic and international financial shocks because the Spanish private sector encountered disruptions on both fronts during our period of interest. In Section 6, we analyze their quantitative significance, and Appendix I examines the effects of disabling them. However, our theoretical findings would remain valid even in the absence of these shocks.

Households' budget constraint: Each period, individual households face a constraint of the form

$$(1-\pi_t)b_t + c_t^T + p_t^N c_t^N = q_t b_{t+1}, + y_t^T + p_t^N y^N + T_t, \tag{3}$$

where T_t is a lump-sum transfer from the government. A positive transfer signifies a bailout, whereas a negative transfer indicates a lump-sum tax. This transfer serves as the primary linkage between households and the government and will be present in all model versions. Having access to this instrument enables the government to directly adjust households' cash-in-hand without introducing additional distortions. Later, we explore the implications of providing the government with an additional tax instrument—a linear tax on private borrowing, denoted as τ_t— used for macroprudential purposes.

Public debt: The government finances itself by issuing, without commitment, a long-term bond($L \geq 0$) on international credit markets *à la* Eaton and Gersovitz (1981). Each period, the government decides whether to default ($d \in \{0, 1\}$) or maintain its access to credit markets by meeting its obligations and issuing new bonds. As in Arellano and Ramanarayanan (2012) and Hatchondo and Martinez (2009), we assume that a bond issued in period t promises in case of repayment a deterministic infinite stream of coupons that decreases at an exogenous constant rate δ. As such, one unit issued in the current period promises to pay a fraction $(1-\delta)$ of all remaining debt each following period. Hence, the debt dynamics can be summarized by

$$L_{t+1} = (1-\delta)L_t + i_t, \tag{4}$$

where L_t is the number of public bonds due at the beginning of period t and where i_t is the bond issuances at t. Similar to the literature, we assume that sovereign debt only takes values in finite and bounded support with $\mathcal{J}$ points. The grid of potential long-term debt positions can be summarized by a vector Λ, where L_j is the j^{th} element; consequently,

$$\Lambda = \Big[L_1, L_2, ..L_{\mathcal{J}}\Big]^T.$$

Default: A sovereign default leads to immediate financial autarky and imposes an additive utility cost that increases with tradable output $\phi(y_t^T)$.[9] We assume that a government in bad standing returns to international credit markets with zero debt with probability θ. It's important to note that sovereign default does not imply default on private debt nor does it result in the exclusion of private agents from financial markets. This stands in contrast to other models featuring both public and private international debt, such as Mendoza and Yue (2009). Empirically, Kalemli-Ozcan et al. (2018), and Bottero et al. (2020) find that although private borrowing declines during a sovereign default crisis, it is still quantitatively significant.

Government's preferences: The government has households' discount factor, and its flow utility at time t is

$$u(C_t) + d_t\big(\epsilon_t^{Def} - \phi(y_t^T)\big) + (1-d_t)\epsilon_t(L_{t+1}),$$

where d_t is the government default decision, C_t is private consumption, $\phi(y_t)$ is the utility cost of default, and ϵ_t is and additive taste shock. The taste shocks are borrowed from Dvorkin et al. (2021) and introduced for computational tractability. We assume that each period the government draws a random vector $\boldsymbol{\epsilon}$ of size $\mathcal{J}+1$ of additive taste shocks. One element of the vector is associated with the choice of default, while the remaining $\mathcal{J}$ elements are associated with each debt choice on Λ in case of repayment. The elements of the vector are labeled

$$\epsilon(L_j) = \epsilon_j,$$
$$\epsilon^{Def} = \epsilon_{\mathcal{J}+1}.$$

The taste shock $\boldsymbol{\epsilon}$ is independent and identically distributed (i.i.d.) over time. Furthermore, we assume that its distribution is a multivariate generalized extreme value with mean m, variance $\sigma^{\epsilon} > 0$, and

[9] Utility losses from default in sovereign debt models are also used in Aguiar and Amador (2013), Bianchi and Sosa-Padilla (2020), and Roch and Uhlig (2018), among others. A common alternative is output costs of default. If the utility function is log over the composite consumption and if output losses from default are proportional to the composite consumption in default, the losses from default would be identical across the two specifications.

correlation parameter p^{ϵ} within Λ.[10]

Government's budget constraint: Each period the government's budget constraint depends on its default status d_t, the public debt dynamics (4), and the lump-sum transfers T_t. It is

$$T_t = (1 - d_t)\Big[Q_t\big[L_{t+1} - (1-\delta)L_t\big] - \delta L_t\Big], \tag{5}$$

where L_t is the long-term public debt at the beginning of period t and where L_{t+1} is the long-term debt at the end. Finally, Q_t is the price at which lenders purchase these bonds, which in equilibrium depends on the government's and household's borrowing decisions and the exogenous shocks.

International lenders: Private and sovereign bonds are traded with a continuum of risk-neutral, competitive foreign lenders. Lenders have access to a one-period risk-free security paying a net interest rate r. The equilibrium price of private bonds is given by the no-arbitrage condition

$$q_t = \frac{\mathbb{E}_t\left[1 - \pi_{t+1}\right]}{1+r}.$$

In equilibrium, investors must be indifferent between purchasing a risk-free security and buying a private bond at price q_t. Since private debt is only held for one period, lenders use the exogenous probability of default one period ahead to price it. Similarly, bond prices for sovereign debt in case of repayment are

$$Q_t = \frac{\mathbb{E}_t}{1+r}\Big[(1 - d_{t+1})\big(\delta + (1-\delta)Q_{t+1}\big)\Big].$$

As before, the no-arbitrage condition implies that investors will purchase government bonds at a price Q_t that compensates them for the risk of default they bear. In case of default, no public debt is recovered. In case of repayment, the payoff is given by the coupon δ plus the market value Q_{t+1} of the non maturing fraction of the bonds next period.

Resource constraints: All debts are denominated in tradables, and market clearing conditions are

[10] For additional details regarding the distribution of taste shocks, see Appendix A. Preference shocks affecting the default decisions are now common in the literature; see, for instance, Arellano et al. (2019), Aguiar et al. (2019), and Aguiar et al. (2020). They are an alternative to the i.i.d. income shocks also encountered in the literature (e.g., Chatterjee and Eyigungor (2012)). In this model, the shocks allow the government to break ties between similar portfolio positions. An interpretation of these shocks is that they capture additional costs or benefits of default, such as the perceptions of policymakers of the costs of default. At the same time, as noted by Dvorkin et al. (2021), provided that the variance of the shocks is small enough, they will have small quantitative consequences in aggregate moments.

$$c_t^N = y_t^N, \tag{6a}$$

$$c_t^T + (1-\pi_t)b_t = y_t^T + q_t b_{t+1} + T_t. \tag{6b}$$

3.2 Baseline unregulated competitive equilibrium

Notation and equilibrium concept: We denote with a prime symbol the end-of-period levels of private and public debt. We focus on the Markov perfect equilibrium. Therefore, the current period decisions of all agents will be functions of payoff-relevant state variables and will take all future policy rules as given. In other words, government default, borrowing, and transfer strategies each period will only depend on current period payoff-relevant states and policies are time-consistent.[11] The government takes as given the best response functions of the other players, households, and foreign lenders, as well as the strategies of future governments that decide policies later on. Importantly, the government considers the general equilibrium effects of its policies on the aggregate choices of the private sector, as well as on all prices.

Timing: The timing of events within the period is :

- If the economy is in good standing ($\varsigma = 1$) it remains so. Otherwise, the economy remains in bad standing ($\varsigma = 0$) with probability $1-\theta$ and $L = 0$.
- The economy enters the period with private debt B and public debt L.
- Shocks are realized, the exogenous state is $s = \{y^T, \kappa, \pi, \epsilon\}$ and the state is $S = \{s, L, B, \varsigma\}$.
- If $\varsigma = 1$, the government makes default d and public debt L' choices.
- If $\varsigma = 0$, the government takes no action the households receive no transfer, (d, L')=$(1, 0)$.
- The aggregate state of the economy incorporating the government's policies is $S_G = \{S, d, L'\}$.
- Facing S_G, households choose consumption and private debt, which determine the aggregate consumption C^T and C^N and the aggregate private debt B'.
- The lenders choose bond schedules Q and q using only the payoff-relevant states.

Policy decisions and best responses: The government's policy decisions are $\boldsymbol{d}(S)$ and $\mathcal{L}'(S)$. The private sector's aggregate best responses are $\mathcal{C}^T(S_G)$, $\mathcal{C}^N(S_G)$, and $\mathcal{B}'(S_G)$. The foreign lenders' best responses are the schedules for public bond $Q(s, L', \mathcal{B}'(S_G))$ and for private bond $q(s)$.

[11] The focus on a Markov perfect equilibrium is important. An environment with strategically defaultable long-term bonds with a government that cannot commit to future debt issuances induces a time inconsistency problem known as debt dilution. The solutions to the recursive, time-consistent problem do not coincide with the solutions to the sequential problem with commitment. For a discussion of policies that remedy debt dilution, see Hatchondo et al. (2016) and Aguiar et al. (2019).

Government: Given the best responses of the private sector and foreign lenders, a government in good standing chooses $\boldsymbol{d}(S)$ and $\mathcal{L}'(S)$ that maximizes the household's welfare subject to the period budget constraint (5) and the resource constraints ((6a) and (6b)). The government's problem is

$$W(S) = \max_{d \in \{0,1\}} [1-d]W^R(S) + dW^D(s, B), \tag{7}$$

where $d = 1$ if the government defaults and $d = 0$ otherwise. If the government repays, its value of repayment is

$$W^R(S) = \max_{L' \in \Lambda} \ u\big(C^T(S_G), C^N(S_G)\big) + \epsilon(L') + \beta \mathbb{E}_s[W(s', L', \mathcal{B}'(S_G))] \tag{8}$$

$$\text{subject to}$$

$$T(S_G) = Q(s, L', \mathcal{B}'(S_G))[L' - (1-\delta)L] - \delta L,$$

$$C^T(S_G) + (1-\pi)B = y^T + q(s)\mathcal{B}'(S_G) + T(S_G),$$

$$C^N(S_G) = y^N.$$

Note that in repayment states, the government's public debt decision affects the value of the transfer the household receives. Thus, the choice of public debt impacts the households' consumption and borrowing decisions. The government internalizes that its borrowing decision affects both the household's choices and the price of public debt.

When the government is in default, its value is

$$W^D\big(s, B\big) = u\Big(C^T(S_G), C^N(S_G)\Big) + \epsilon^{Def} - \phi(y^T) + \beta \mathbb{E} W^D \tag{9}$$

$$\text{subject to}$$

$$C^T(S_G) + (1-\pi)B = y^T + q(s)\mathcal{B}'(S_G),$$

$$C^N(S_G) = y^N.$$

$$\mathbb{E}W^D = \mathbb{E}_s\Big[\theta W\big(s', 0, \mathcal{B}'(S_G), \varsigma = 1\big) + (1-\theta)W^D\big(s', \mathcal{B}'(S_G)\big)\Big].$$

While in default, the government loses access to credit markets, and the transfer is zero. Nevertheless, households still maintain access to financial markets and are still liable for their obligations. Consequently, a sovereign default can still leave the economy highly leveraged, albeit in private bonds.

The solution to the government's problem yields decision rules for default $\boldsymbol{d}(S)$ and public debt $\mathcal{L}'(S)$, which in turn determine the transfers $T(S_G)$

$$T(S_G) = (1 - \boldsymbol{d}(S)) \times \Big(Q(s, \mathcal{L}'(S), \mathcal{B}'(S_G))[\mathcal{L}'(S) - (1-\delta)L] - \delta L\Big), \tag{10}$$

Households: The households make decisions based on their current level of individual debt b and the aggregate state of the economy when they act S_G. The aggregate state consists of the exogenous shocks s, the initial level of government debt L, the current level of aggregate private debt B, and the decisions made by the government in the current period regarding default d and public debt L'. Households are competitive, and as such they take all prices and aggregate laws of motion as given: the price of nontradables $p^N(S_G)$, the equilibrium price of private bonds $q(s)$, the government's current and all future borrowing decisions $\mathcal{L}'$ and default decisions $\boldsymbol{d}$,[12] and transfers T. Under rational expectations, households predict future states using the perceived law of motion of aggregate private debt $\mathcal{B}'$. The households' optimization problem in recursive form is

$$V(S_G, b) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + \beta \mathbb{E}_s[V(S'_G, b')] \tag{11}$$

subject to

$$c^T + p^N(S_G)c^N + (1-\pi)b = y^T + p^N(S_G)y^N + q(s)b' + T,$$

$$q(s)b' \leq \kappa[p^N(S_G)y^N + y^T],$$

$$(T, B', L') = (T(S_G), \mathcal{B}'(S_G), \mathcal{L}'(S)),$$

$$S'_G = (s', L', B', \boldsymbol{d}(s', L', B'), \mathcal{L}'(s', L', B')).$$

In equilibrium, $p^N(S_G)$ is the price of nontradables, and $q(s)$ is the price of private bonds. The solution to the household problem yields decision rules for individual bond holdings $\hat{b}'(S_G, b)$, tradable consumption $\hat{c}^T(S_G, b)$, and nontradable consumption $\hat{c}^N(S_G, b)$. The household optimization problem induces a mapping from the perceived law of motion for aggregate bond holdings, $\mathcal{B}'(S_G)$, to an actual law of motion, given the representative agent's choice $\hat{b}'(S_G, B)$. In a rational expectations equilibrium, these two functions must coincide. The same is true for the laws of motion of aggregate consumption in the economy $\{C^i(s, L, B)\}_{i=T,N}$.

The solutions to the households' problem solve the optimality conditions that include the budget constraint (3), the credit constraint (2), and the first-order conditions. In particular, the households' intratemporal optimality condition pins down the equilibrium price of nontradables:

$$p^N(S_G) = \frac{1-\omega}{\omega}\left(\frac{C^T(S_G)}{y^N}.\right)^{\eta+1}. \tag{12}$$

Condition (12) equates the marginal rate of substitution between tradable and nontradable goods to their relative price. Thus, in equilibrium, the price of nontradables is increasing in c^T.A pecuniary

[12] For concision's sake, we equate in the discussion the solutions to the current government policy functions with the strategies of future governments. This equality holds in a Markov perfect equilibrium. Alternatively, one could impose this equality as an equilibrium condition, as in Bianchi and Mendoza (2018). Finally, if the government is in bad standing and does not reenter its default and borrowing policies are pinned down $(\boldsymbol{d}(S), \mathcal{B}'(S_G))) = (1, 0)$.

externality arises in this problem because this equilibrium price affects the value of collateral (2) and therefore the level of borrowing in some states. As a result, a reduction in c^T causes in equilibrium a reduction in the collateral value (2). In states where the credit constraint binds, this reduction triggers a financial amplification mechanism, whereby a drop in consumption induces a contraction in private borrowing, which in turn drives consumption further down. Since a fiscal transfer will at least in part be consumed by the households, bailouts can mitigate this amplification mechanism, which creates an incentive for the government to issue public debt during crises.

Lenders: The risk-neutral, competitive foreign lenders use the decision rules of current and future governments and households to price the bonds. The solution to the problem of the lenders yields the bond price schedule for private debt,

$$q(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r}, \tag{13}$$

and the bond price schedule for public debt,

$$Q(s, L', B') = \frac{1}{1+r} \times \mathbb{E}_s\left[\left[1-d'\right] \times \left[\delta + (1-\delta)Q\left(s', L'', B''\right)\right]\right], \tag{14}$$

where

$$(B'', L'', d') = \left(\mathcal{B}'(s', L', B'), \mathcal{L}'(s', L', B'), \boldsymbol{d}(s', L', B')\right).$$

The lenders price the debt contracts based on their expectations of future defaults and new issuances of debt. When pricing private debt, the only payoff-relevant state is the exogenous shock s. In contrast, when pricing public debt, the payoff-relevant states for the lenders also include the end-of-period levels of private B' and public debt L'. Note that both the levels and composition of debt are important because they affect the future governments' default and public debt issuance decisions.

Definition 1. *A competitive unregulated Markov equilibrium is a set of value functions $\{V, W, W^R, W^D\}$, policy functions for the private sector $\{\hat{b}, \hat{c}^T, \hat{c}^N\}$, policy functions for the public sector $\{\boldsymbol{d}, \mathcal{L}'\}$, a pricing function for nontradable goods p^N, pricing functions for public debt Q and private debt q, and perceived laws of motion $\{\mathcal{B}', C^T, C^N, Q\}$ such that*

1. *Given prices $\{p^N, q\}$, government policies $\{\boldsymbol{d}, \mathcal{L}'\}$, and perceived law of motion $\mathcal{B}'$, the private policy functions $\{\hat{b}', \hat{c}^T, \hat{c}^N\}$ and value function V solve the household's problem (11).*

2. *Given bond prices $\{Q, q\}$ and aggregate laws of motion $\{\mathcal{B}', C^T, C^N\}$, the public policy functions $\{\boldsymbol{d}, \mathcal{L}'\}$ and value functions W, W^R, and W^D solve the Bellman equations (7)–(8).*

3. *Households' rational expectations: perceived laws of motion are consistent with the actual laws of motion $\{\mathcal{B}'(S_G) = \hat{b}'(S_G, B), C^T(S_G) = \hat{c}^T(S_G, B), C^N(S_G) = \hat{c}^N(S_G, B)\}$.*

4. The private bond price function $q(s)$ satisfies (13).

5. Given public $\{\boldsymbol{d}, \mathcal{L}'\}$ and private $\{\mathcal{B}'\}$ policies, the public bond price $Q(s, L', B')$ satisfies (14).

6. Goods market clear:

$$\begin{aligned} C^N(S_G) =& y^N, \\ C^T(S_G) + (1-\pi)B =& y^T + q(s)\mathcal{B}'(S_G) + \\ & \Big\{1 - \boldsymbol{d}(S)\Big\}\Big\{Q(s, \mathcal{L}'(S), \mathcal{B}'(S_G))\Big[\mathcal{L}'(S) - (1-\delta)L\Big] - \delta L\Big\}. \end{aligned}$$

3.3 Recursive social planner's problem

Next, we formulate the problem of a social planner in the same environment. The formulation is similar to the "primal approach" to optimal policy analysis. The planner chooses aggregate allocations subject to resource, implementability, and collateral constraints. Note that the planner does not set prices and instead takes the pricing functions that solve the lenders' problem as given. However, the planner internalizes how their consumption and borrowing decisions affect all general equilibrium prices. As such, the planner behaves like a strategic player and not competitively as the households do in the previous subsection. Therefore, the equilibrium price of nontradable goods (p^N) and bonds (q, Q) will enter the problem as implementability constraints. As before, the focus is on the Markov perfect stationary equilibrium. We assume that the planner cannot commit to future policy rules, including future defaulting and borrowing decisions. Consequently, the planner chooses current period allocations, taking as given the strategies of future planners.

The social planner's (SP) optimization problem consists of maximizing the utility of the households (1) subject to the credit constraint (2), the resource constraints ((6a) and (6b)), and equilibrium prices ((12), (13), and (14)). Denote $\mathcal{L}'^{SP}$ and $\mathcal{B}'^{SP}$ as the public and private borrowing decisions, respectively. Let $\boldsymbol{d}^{SP}$ be the default decisions of future planners that the current SP takes as given. The planning problem is[13]

$$W^{SP}(s, L, B) = \max_{d \in \{0,1\}} [1-d] W^{SP,R}(s, L, B) + d W^{SP,D}(s, B), \tag{15}$$

[13]The equilibrium price of nontradables (12) and the resource constraint of nontradables (6a) are already incorporated in this formulation. The price of public bonds Q^{SP} is the equilibrium best response of risk-neutral, competitive lenders. Moreover, the household budget constraint is satisfied by Walras's law.

where the default value of the planner $W^{SP,D}(s,B)$ is

$$
\begin{aligned}
W^{SP,D}(s,B) &= \max_{c^T,B'} u\left(c^T,y^N\right) - \phi(y^T) + \epsilon_{Def} + \beta\mathbb{E}_s\left[\theta W^{SP}(s',0,B') + (1-\theta)W^{SP,D}(s',B')\right],\\
c^T + B(1-\pi) &= y^T + q^{SP}(s)B',\\
q^{SP}(s)B' &\leq \kappa\left(\frac{1-\omega}{\omega}\left(\frac{c^T}{y^N}\right)^{\eta+1} y^N + y^T\right),\\
q^{SP}(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r}.
\end{aligned}
\tag{16}
$$

And the value of the planner under repayment $W^{SP,R}(s,L,B)$ is

$$
\begin{aligned}
W^{SP,R}(s,L,B) &= \max_{c^T,B',L'\in\Lambda} u\left(c^T,y^N\right) + \epsilon(L') + \beta\mathbb{E}_s[W^{SP}(s',L',B')],\\
c^T + B(1-\pi) + \delta L &= y^T + q^{SP}(s)B + Q^{SP}(s,L',B')[L' - (1-\delta)L],\\
q^{SP}(s)B' &\leq \kappa\left(\frac{1-\omega}{\omega}\left(\frac{c^T}{y^N}\right)^{\eta+1} y^N + y^T\right),\\
q^{SP}(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r},\\
Q^{SP}(s,L',B') &= \frac{\mathbb{E}_s\left[\left[1-\boldsymbol{d}^{SP}(s',L',B')\right]\times\left[\delta + (1-\delta)Q^{SP}\left(s',\mathcal{L}^{SP\prime}(s',L',B'),\mathcal{B}^{SP\prime}(s',L',B')\right)\right]\right]}{1+r}.
\end{aligned}
$$

Like the government, the planner chooses aggregate private debt L'. In contrast to the government in the baseline version, the planner also directly controls the level of aggregate private borrowing B'. The planner's decisions take into account the effect of these choices on the price of nontradables (12), the value of collateral (2), and the price of public debt (14).

Definition 2. *A Markov stationary socially planned equilibrium is a set of value functions* $\{W^{SP}, W^{SP,R}, W^{SP,D}\}$*, policy functions for allocations* $\{C^{SP,T}, C^{SP,N}, \mathcal{L}'^{SP}, \mathcal{B}'^{SP}\}$*, defaulting* $\boldsymbol{d}^{SP}$*, and pricing functions for public* Q^{SP} *and private* q^{SP} *debt that solve (15) given conjectured future policies* $\{C^{SP,T}, C^{SP,N}, \mathcal{L}'^{SP}, \boldsymbol{d}^{SP}\}$

3.4 Decentralization with macroprudential policies

Finally, we consider another version of the decentralized model where the government gains access to state-contingent linear taxes on private borrowing. We show that the Markov competitive equilibrium allocation solves the planner's problem presented in the previous subsection. The households' budget constraint (3) becomes

$$
(1-\pi_t)b_t + c_t^T + p_t^N c_t^N = q_t(1-\tau_t)b_{t+1}, + y_t^T + p_t^N y^N + T_t,, \tag{17}
$$

where τ_t is the tax rate on private borrowing. The introduction of taxes does not modify the credit constraint (2). As with all other government policies, taxes on private debt are taken as given by households. At the same time, the government still taxes the households using lump sums. The budget constraint (5) is now

$$T_t = (1 - d_t)\Big[Q_t\big[L_{t+1} - (1-\delta)L_t\big] - \delta L_t\Big] + \tau_t q_t B_{t+1}. \tag{18}$$

Note that the government can tax private debt and use lump-sum transfers while in default. Appendix A provides a complete recursive formulation and characterization of the decentralized equilibrium with taxes.

Proposition 1. *The socially planned equilibrium allocation can be decentralized with a state-contingent tax on debt that satisfies*

$$1 - \boldsymbol{\tau}(s, L, B) = \frac{\mu^{SP}(s, L, B)q^{SP}(s) + \beta\mathbb{E}_s\Big[(1-\pi')\Big(u_T^{SP}(C^{SP,T}(s', L', B'), y^{N'})\Big)\Big]}{q^{SP}(s)u_T(C^{SP,T}(s, L, B), y^N)}, \tag{19}$$

where μ^{SP} corresponds to the Lagrange multiplier associated with the credit constraint in the planner problem (15).

Proof: See Appendix B.

The proof is done in two steps. First, we show that the planning problem is equivalent to a relaxed version of the competitive equilibrium with taxes. Second, we show that solutions to the planning problem are sufficient to construct policies that satisfy the additional constraints of the competitive equilibrium problem with taxes.

3.5 The two sources of private overborrowing in the baseline economy

This subsection explains the intuition behind the main difference between the baseline and planned economies. Consider the intertemporal optimality conditions of the households in the baseline problem (11),

$$q(s)u_T(C^T(S_G)) = \beta\mathbb{E}_s[(1-\pi')u_T(C^T(S'_G))] + \mu(S_G)q(s), \tag{20}$$

$$0 \leq \kappa(p^N(S_G)y^N + y^T) - q(s)\mathcal{B}'(S_G) \quad \text{with equality if } \mu(S_G) > 0, \tag{21}$$

where $u_T(.)$ is shorthand notation for $\frac{\partial u}{\partial c}\frac{\partial c}{\partial c^T}$, the marginal utility of the tradable consumption, and where μ is the Lagrange multiplier on the credit constraint.[14] The prime notation denotes future

[14]These expressions are obtained by assuming that the policy and value functions are differentiable and then applying the standard envelope theorem to the first-order conditions of the household problem while assuming that rational expectations hold. Appendix A characterizes in detail the optimality conditions of the planning problem.

values. Equation (20) is the household's Euler equation for private debt, and equation (21) is the complementary slackness condition. If $\mu > 0$, the marginal utility benefits from increasing tradable consumption today exceed the expected marginal utility costs from borrowing one unit of private debt and repaying next period. The main difference between the baseline and the planner's economy is in the private borrowing decision. Thus, we compare the Euler equations of private bonds for the two problems rearranging the terms to facilitate the comparison. Using the same notation as before, we rearrange equation (20), and its counterpart from the planned economy (SP) as:

$$
\begin{aligned}
qu_T - \left\{\beta\mathbb{E}[(1-\pi')u_T'] + \mu q\right\} &= 0, \\
qu_T^{SP} - \left\{\beta\mathbb{E}[(1-\pi')u_T^{SP'}] + \mu q\right\} &= \left\{\beta\mathbb{E}[(1-\pi')\mu'\psi'] - q\mu\psi\right\} - \left\{Q_{B'}^{SP}[L' - (1-\delta)L]\right\}\left\{u_T^{SP} + q\mu\psi\right\}.
\end{aligned}
$$

The planner's Euler equation contains two additional objects, ψ and $Q_{B'}^{SP}$, that reflect the two sources of overborrowing.[15] First, the term:

$$
\psi = \kappa(1+\eta)\frac{(1-\omega)}{\omega}\left(\frac{c^T}{y^N}\right)^{\eta},
$$

is the marginal impact of an extra unit of tradable consumption on the value of collateral. An additional unit of private debt increases the relative price of nontradables in the present period as per equation (12); however, it concurrently diminishes the relative price of nontradables in the subsequent period when the debt is repaid. Hence, this term appears in front of the Lagrange multiplier associated with the current period credit constraint (μ), but also in expectation with the next period's credit constraint (μ'). The second term,

$$
Q_{B'}^{SP} = \frac{\partial Q^{SP}(s, L', B')}{\partial B'},
$$

is the marginal effect of an additional unit of private debt on the price of public debt. Since private debt affects future government policies lenders factor them in when pricing government debt. The two terms in the right-hand side of the planner's Euler equation encapsulate the two sources of overborrowing. We find that both effects are quantitatively significant for our results.

First source of overborrowing The term,

$$
\left\{\beta\mathbb{E}_s[(1-\pi')\mu^{SP'}\psi'] - q\mu^{SP}\psi\right\},
$$

[15] Throughout the paper we refer to these terms as the two sources of overborrowing, as in our calibrated equilibrium, they will both contribute to higher private debt in the baseline model relative to the planner. However, we acknowledge that they could also result in private *underborrowing* under different parameter settings. For the first term, underborrowing is discussed in Schmitt-Grohé and Uribe (2021), and, for the second source in Kim and Zhang (2012).

appears in Bianchi (2011) and models with an equilibrium price in the collateral credit constraint. It captures what the literature calls the pecuniary externality of private debt. Unlike households in the baseline model, the planner internalizes that additional borrowing diminishes future borrowing capacity by devaluing nontradables. Consequently, the planner opts for reduced borrowing. It is crucial to note that disregarding this effect is rational from the standpoint of an individual household. Each household is small and lacks control over aggregate borrowing, thus its borrowing decisions hold no sway over aggregate prices.

Second source of overborrowing The term,

$$\left\{Q_{B'}^{SP}[L' - (1-\delta)L]\right\}\left\{qu_T^{SP} + q\mu^{SP}\psi\right\},$$

appears in models of sovereign debt with multiple assets such as Arellano and Ramanarayanan (2012) and Hatchondo et al. (2016), yet it is directly associated with overborrowing solely in models where borrowing decisions are decentralized, such as Kim and Zhang (2012). In equilibrium, an additional unit of private debt affects the price of public debt through two channels. Firstly it changes default rules in the subsequent period, as both repayment and default values hinge on the initial level of private debt. Secondly, it impacts the expected issuances of public debt, thereby the degree of public debt dilution.[16] The effect of private debt on default sets and public borrowing is explored in detail sections 7 and 8.1.

4 Calibration

Numerically, we solve the baseline problem using time iteration for the private economy and value function iteration for the government. The planner's problem is solved with value function iteration.[17]

The baseline version of the model is calibrated at the annual frequency using Spanish macroeconomic data from 1999 to 2012. The model will be validated using data from 2008 to 2015, thus including some out-of-sample data. We assume that Spain was at the ergodic distribution of the baseline version of the model during this period.

The choice of calibration period is informed by our modeling assumption and institutional reasons. From a modeling perspective, we aim to have a quantitative sovereign debt model that simultaneously captures the low average level of spreads observed in the decade preceding the crises, and the large spike in spreads observed in 2012. Moreover, the start and end dates also coincide with a specific institutional framework that changed significantly before and after those dates. The starting year is

[16]This second channel would not be relevant for the price of current public debt when public debt is also one period. See appendix I for a discussion about the effect of the maturity structure of public debt.

[17]More details regarding the numerical solution methods are described in Appendices D and E. Details about our data sources can be found in Appendix C

chosen to coincide with the creation of the Eurozone. Before this, most Spanish public debt was in domestic currency, and therefore its nominal value was subject to government choices. The end year of 2012 is also significant because European-wide policies were introduced where that year in response to the crisis. Some of these policies conflict with some of the fundamental assumptions underlying the baseline version of the model. Although Spain had implemented countercyclical prudential policies for its domestic banking sector in 1999, up until 2012 there were no systematic controls on private international borrowing within the European Union. This changed in June 2012, when European heads of state proposed the creation of the Single Supervisory Mechanism (SSM) to supervise bank debt within the union. By 2014, the Bank of Spain had transferred a substantial portion of its supervisory powers to the SSM.[18]

Given that the baseline version of the model assumes no restrictions on international private debt, we end the calibration to the year of their introduction. In this section, we detail our functional for assumption and present the parameters that we estimate outside of the model in Table 1, as well as the calibrated parameters and the targets in Table 2.

Functional forms: The utility function is of the constant relative risk aversion (CRRA) form on the composite CES good:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} \quad \text{with } \sigma > 0.$$

The default utility cost is parameterized as follows:

$$\phi(y^T) = \max\{0, \phi_0 + \phi_1 \ln y^T\}.$$

As Arellano (2008) and Chatterjee and Eyigungor (2012) discuss, a nonlinear specification of the default costs allows the model to reproduce the mean and standard deviation of spreads in the data. We follow Bianchi et al. (2018) in specifying the default cost function in terms of utility to avoid drops in private borrowing capacity during defaults.

Parameters estimated outside of the model: The risk aversion σ and elasticity of substitution between tradables and nontradables $1/(1+\eta)$ are set at standard values. The preference parameter ω is chosen to replicate the share of tradable GDP in the data, which is 40%.[19] Since the average tradable

[18] In addition, in June of 2012, European leaders also agreed to allow the European Stability Mechanism to offer direct help to Spanish banks. Finally, one month later, in July 2012, then-president of the European Central Bank (ECB) Mario Draghi famously signaled the commitment of the institution to do "whatever it takes to preserve the euro." That statement was interpreted at the time as a commitment from the ECB to buy Eurozone public bonds from distressed countries. For a discussion of how beliefs can be crucial for sovereign default incentives, see Aguiar et al. (2020), and Paluszynski (2023).

[19] To compute the model counterpart of this object at the ergodic distribution, we use the mean value of external private liabilities $\bar{b}$ and external public liabilities $\bar{L}$ at their targeted values.In the baseline calibration described below, $\bar{b} = 0.43$ and $\frac{\delta}{1+\frac{1-\delta}{1+r}}\bar{L} = .14$. The value of ω is then set so that $\frac{\bar{p}^N y^N}{\bar{p}^N y^N + y^T} = 0.60$, where $\bar{p}^N = \frac{1-\omega}{\omega}\frac{y^T - r\bar{b} - \delta r \bar{L}}{y^N}$. Tradable GDP is computed using the value-added shares of agriculture, manufacturing, and tradable services, more details are given in Appendix C.

Table 1: Parameters estimated outside of the model

Description	Parameter	Value
Risk aversion	σ	2.0
Elasticity of substitution	$1/(1+\eta)$	.83
Share of tradables	ω	.39
Persistence of tradables	ρ^y	.75
Volatility of tradables	σ^y	.010
Mean private default rate	$\bar{\pi}$	.026
Persistence private default rate	ρ^π	.91
Volatility private default rates	σ^π	.32
Risk free interest rate	r	.025
Duration of long-term bonds	δ	.14
Probability of reentry	θ	.33

Note: The risk aversion and elasticity of substitution between tradables and nontradables are standard in the literature. The share of tradables is the average share of value added of agriculture, manufacturing, and tradable services of GDP. The risk-free rate is the average yield of one-year German treasury bonds. The duration parameter is chosen to match the average bond duration of six years of Spanish bonds. The tradable income and private default shock parameters are estimated by fitting a first-order autoregressive process on the logs of the tradable share of GDP and share of nonperforming gross loans, respectively. All public bond and yield data are from 1999 to 2012, and the processes for tradable income and nonperforming loans are estimated using the longest available series. The data source for bond yields and nonperforming loans is Bloomberg, and the sectoral GDP series are taken from Eurostat. For details, see data Appendix C. The period of market exclusion is taken from Richmond and Dias (2009).

and nontradable endowments are one, this yields $\omega = .39$.

The risk-free interest rate, r, is set to the average yield of the one-year German treasury bill over the calibration period, $r = 2.5\%$. One-year bonds are chosen as a benchmark to reproduce the maturity of the short-term private bond in the model. The duration parameter δ is chosen so that the average duration in the model corresponds to the average maturity of Spanish bonds in the data, 6 years. This calculation is in line with previous estimates of Spanish bond maturity, as those from Hatchondo et al. (2016) and Bianchi and Mondragon (2018).[20] The implied duration is then $\delta = .14$. The reentry probability after default is θ is set to .33 following Richmond and Dias (2009).

Exogenous shocks: There are three types of shocks in our computational calibration: income shocks (y^T), financial shocks (κ and π), and taste shocks (ϵ). We follow Bianchi et al. (2016) and find that including shocks to the endowment of tradables, y^T, while keeping the endowment of nontradable, y^N, constant and equal to one is sufficient to generate business cycle statistics that are consistent with the data in terms of volatility of aggregate output and its correlation with consumption and re-

[20]The Macaulay definition of duration of a bond given the coupon structure of the model is

$$M = \frac{1 + \bar{i_L}}{\delta + \bar{i_L}},$$

where $\bar{i_L}$ is the constant per-period yield delivered by a long-term bond held to maturity (forever) with no default. At the calibrated values, this corresponds to the targeted spread plus the risk-free rate.

duces the state space.[21] Since the focus is on fluctuations around the business cycle, we estimate the tradable process using the cyclical component of linearly detrended tradable GDP for Spain. The estimated values for persistence and volatility are $\rho^y = .75$ and $\sigma^y = .01$, respectively. The recursive specification is

$$\ln y_t^T = \rho^y \ln y_{t-1}^T + \varepsilon_t^y \quad \text{with } \varepsilon_t^y \sim N(0, \sigma^y).$$

The domestic financial shock, π, corresponds to the exogenous share of private bonds defaulted on each period. Quantitatively, this shock will help us make private debt more volatile than total and public debt, a salient fact in the data.[22] We again assume it follows a log-normal AR(1) process of the form:

$$\ln \pi_t = (1 - \rho^\pi)\bar{\pi} + \rho^\pi \ln \pi_{t-1} + \varepsilon_t^\pi \quad \text{with } \varepsilon_t^\pi \sim N(0, \sigma^\pi).$$

The parameters of this process are estimated using the gross share of nonperforming loans as a percentage of total loans. The estimation yields an average private default rate $\bar{\pi} = 2.1\%$, a persistence parameter $\rho^\pi = .91$, and a volatility $\sigma^\pi = .33$.

The international financial shock κ, captures the share of market value of output that can be pledged as collateral. This shock is included to capture, in a reduced form, the fact that the Spanish crises coincided with financial turmoil around other Southern European countries, thus this shock captures fluctuations in private borrowing that are not driven by domestic factors nor spreads. As with the domestic financial shock, this parameter will be important to increase the volatility of private debt relative to the data. Since the credit constraint is only occasionally binding, this shock has unfortunately no direct data counterpart, but it can be partially identified by the average level of private debt and its volatility. Consequently, we assume that it follows a first-order normal AR(1) process of the form

$$\kappa_{t+1} = (1 - \rho^\kappa)\bar{\kappa} + \rho^\kappa \kappa_t + \varepsilon_t^\kappa \quad \text{with } \varepsilon_t^\kappa \sim N(0, \sigma^\kappa).$$

For simplicity, we assume that the persistence parameter coincides with the persistence of tradable income $\rho^\kappa = \rho$ while the mean ($\bar{\kappa}$) and volatility parameters (σ^κ) are calibrated internally. This yields values of $\bar{\kappa} = .48$ and $\sigma^\kappa = .02$. While we are able to approach the volatility of total debt and public debt very well, targeting the very large standard deviation of private debt observed in the data 10 percent of output is more challenging. The two financial shocks help the model achieve a volatility of private debt at the ergodic distribution of 7.0 of output.

[21] In Appendix G we present the untargetted business cycle statistics. In Appendix I.3 we allow for shocks to the endowment of nontradables perfectly correlated with the shocks to tradables. Our main quantitative findings are robust to this change. Note however that this assumption implies that we will not be able to trace the path of aggregate output. Our focus will be on the cyclical component of tradables. We follow a reduced form approach to output and see our model as a complement to sovereign debt models of the European debt crisis with endogenous production, such as Arellano et al. (2019), and with regimes of output growth, such as Paluszynski (2023).

[22] In Appendix I we explore the effects of estimating the model without the domestic and international financial shocks. Without these shocks, the volatility of private debt would be lower.

Table 2: Calibrated parameters

Description	Parameter	Value	Moment	Target	Model
Discount factor	β	.95	Avg. total debt	58	59
Vol. taste shock	σ^{ϵ}	8e-3	Vol. total debt	4.8	4.9
Corr. taste shock	p^{ϵ}	.91	Vol. public debt	6.4	6.6
Avg. financial shock	$\bar{\kappa}$	.48	Avg. private debt	43	43
Vol. financial shock	σ^{κ}	.020	Vol. private debt	10	7.0
Default cost	ϕ_0	.10	Avg. spread	.73	.81
Default cost	ϕ_1	1.4	Vol. spread	.89	.73

Note: Total debt and private debt are computed using the international investment position presented in Section 2. Spreads correspond to the difference between the interest rate paid by Spanish six-year bonds and their German equivalents. All moments are computed using data from 1999 to 2011. For additional details, see Appendix C.

The taste shocks, ϵ, are included for computational purposes and their calibrated parameters are identified by the volatility of total and public debt. As Dvorkin et al. (2021) we find a low volatility in the baseline calibration ($\sigma^{\epsilon} = 8e - 3$). We also calibrate the correlation parameter $p^{\epsilon} = .95$ to match the standard deviation of public debt.[23] The model replicates the lower volatility of total debt relative to its two sub-components.

Debts and spreads: We follow the sovereign debt literature and use the discount factor β, as well as the exogenous default costs,ϕ_0 and ϕ_1, to match the average level of total debt, the average level of public debt, as well as the first and second moments of the average interest rate spread paid on public debt. We obtain a discount factor of .95, in line with with sovereign debt literature at annual frequencies.

The parameters associated with the default costs ϕ_0 and ϕ_1 are measured in the data using the difference in returns between the average Spanish six-year bond and the average German bond of the same maturity. The targeted moments are the average and the standard deviation of this spread, and their model counterparts are the average and standard deviation of the spread of the long-term bond L_t. To compute the sovereign spread in the model that is implicit in a bond price Q, we use the definition of the constant per-period yield. Given the coupon structure, the yield satisfies

$$Q = \sum_{j=1}^{\infty} \delta \frac{(1-\delta)^{j-1}}{(1+\bar{i_L})^j}.$$

The average targeted spread is 0.73% with a standard deviation of 0.89%, which implies values for the default cost parameters of $\phi_0 = .10$ and $\phi_1 = 1.4$. These target spreads are very low when compared to the literature. This is because we computed using the entire 1999-2012 data. Other quantitative sovereign debt models focused on Spain, such as those of Hatchondo et al. (2016) and Bianchi and

[23]The mean of the taste shocks is irrelevant for their quantitative properties and is selected to achieve numerical tractability. More details can be found in Appendix D.

Mondragon (2018), focus instead on 2011-2015 and, consequently, target a higher spread. Since the paper aims to study the link between the buildup of private debt during the years 1999-2007, when the interest rate spread of government debt was very close to zero, and the subsequent sovereign debt crisis, it is important for the model to simultaneously match both low averages and the large spikes observed during the crisis.

5 Results: aggregate moments at the ergodic distribution

Table 3 presents the quantitative results of the paper. The table shows the values of the first and second moments in the data and at the ergodic distributions of the baseline and the socially planned economies. The baseline version of the model is calibrated to match the moments from the data; the socially planned economy is not. Instead, we use the calibrated parameters of the baseline to solve this version of the model. The average private debt for the social planner is 35% of output, whereas it is 43% in the baseline case. This difference of 8% of output is our estimate of the total amount of excessive private debt in Spain in the lead-up to the crisis. Moreover, the baseline economy accumulates on average more public debt than the planned economy, around 1% of output.

In the bottom half of the table, we compute the implications of these portfolio decisions for four measures of crises and aggregate well-being. Specifically, the probability of a binding credit constraint, the probability of a financial crisis, the probability of a sovereign default, and a measure of welfare gains. The credit constraint binds more frequently under the baseline which in turn will also make bailouts more frequent. We follow the literature and define a financial crisis as an episode of significant deleveraging in private debt, more than two standard deviations below the mean of the current account of the private sector. Under this definition, we find that excessive private borrowing increases the incidence of financial crises by .40 p.p. on average.

Furthermore, Table 3 shows that private overborrowing in the baseline economy increases sovereign risk. The probability of default increases from .9% to 1.6%. This is also reflected in the interest rate spreads on public debt. On average the planner pays spreads that are half of what the government pays in the baseline economy. The reduction in the spreads, however, can also be driven by the fact that the planner borrows less in general. The implications of private overborrowing for sovereign risk are explored in detail in section 8.1.

Finally, Table 3 shows the welfare gains of moving from the baseline to the planned economy. The welfare gains, .26%, are calculated as the proportional increase in consumption for all possible future states that would make the households indifferent between staying in the baseline and moving to the centralized equilibrium. This measure explicitly incorporates the cost of lower consumption in the transition to the ergodic distribution of the planned economy. By comparison, in Bianchi (2011), the welfare gains from correcting the overborrowing externality are around .13%. The welfare gains here are larger because optimal private debt management also decreases the probability of experiencing the

Table 3: Baseline and social planner moments at the ergodic distribution

Moments (in %)	Data	Baseline	Social planner
Total debt	58	59	50
Private debt	43	43	35
Public debt	16	16	15
Mean spread	.73	.81	.38
Volatility debt	4.8	4.9	4.2
Volatility private debt	10	7.0	9.7
Volatility public debt	6.4	6.6	8.9
Volatility spread	.89	.73	.38
Probability of a binding constraint	-	5.9	1.0
Probability of a financial crisis	-	2.5	2.1
Probability of default	-	1.6	.90
Welfare gains	-	-	.26

Note: All calibrated parameters are kept constant in the computation of the socially planned economy. The debt levels are expressed as percent of output. The interest rates, the probabilities, and the welfare gains are in percent. Volatilities are standard deviations. A financial crisis is defined as an episode in which the current account of the private sector contracts by more than two standard deviations below its long-run mean. The probability of default corresponds to the percent of time at the ergodic that the economy is in default. Welfare gains are calculated as the proportional increase in permanent consumption under the baseline. Debt levels in the data are calculated using the international investment positions. More details are provided in Appendix C.

deadweight losses of sovereign default and the use of short-term private debt offsets the inefficiencies stemming from sovereign debt dilution.[24]

6 Validation : The 2012 debt crisis

This subsection uses the data to validate the model's capability to produce spreads akin to those witnessed in Spain between 2008 and 2015 when confronted with the same shocks. We use the data to retrieve the tradable income shock (y^T), and the two financial shocks (π and κ). Subsequently, we use the policy functions of the model to see what the evolution of private debt, public debt, and interest rate spreads would have been. We also juxtapose, against the baseline model and the data, the counterfactual dynamics of a socially planned economy.

The tradable income shock, y^T, is taken directly from the Spanish tradable GDP data. Similarly, the financial shock associated with default on private debt, π, is set to match exactly the data on gross nonperforming loans during this period. The taste shocks, ϵ_t, are all set to zero. The second financial

[24]Additional comparisons of aggregate moments of the model to those of nested variants of the model (with no public debt and no private debt) can be found in Appendix I. In addition to the targeted moments presented in Table 3, the quantitative performance of the model for untargeted business cycle moments is presented in Appendix G. The model successfully approximates the volatility of consumption, the current account, and the trade balance, but overestimates the volatility of output. Moreover, the baseline model correctly predicts the sign of the correlations between output and consumption, output and the current account, output and the spread on public debt, and the public debt level and the spread on public debt.

shock, however, κ, is unobserved in the data. To circumvent this issue, we follow Bocola and Dovis (2019) and apply the particle filter method to retrieve it from the data using the model. Additional details about the particle filter method can be found in appendix H; here we present a summary of the methodology.

The baseline model defines a nonlinear state-space system:

$$\boldsymbol{Y}_t = g(\boldsymbol{S}_t) + e_t,$$
$$\boldsymbol{S}_t = f(\boldsymbol{S}_{t-1}, \varepsilon_t),$$

where $\boldsymbol{S}_t = [L_t, B_t, y^T_{t-1}, \pi_{t-1}, \kappa_{t-1}]$ is the state vector and where ε_t is the vector collecting all the innovations in the three structural exogenous shocks (y^T, π, κ). The vector of observables, $\boldsymbol{Y}_t$, includes average private and public debt as a share of GDP, detrended tradable output, the share of non-performing loans, and the interest rate spreads on public bonds.[25] The vector e_t represents uncorrelated Gaussian measurement errors and is equal to the difference between the data aggregates $\boldsymbol{Y}_t$ and their model counterparts $g(\boldsymbol{S}_t)$. The functions $g(\cdot)$ and $f(\cdot)$ come from the calibrated numerical solutions of the baseline model. The realizations of the state vector are estimated by applying the particle filter to this system of equations and data from 2008 to 2015.[26] The process yields a path of financial shocks and a set of initial endogenous states. We feed these shocks into the social planner's policy functions to generate the allocations of debts and spreads that would have emerged under optimal policies. Note that the social planner functions are not used to estimate the financial shock κ. Finally, we also compute the implied tax on private debt that implements the planner allocations in a competitive equilibrium. Figure 4 summarizes the results of this exercise.

The shocks: Panel (a) to (c) of figure 4 depict the shocks utilized in both models while also highlighting the importance of selecting this specific set of shocks that informed our modeling choices. Panel (a) shows the income shocks, directly sourced from the cyclical component of output in the data, showcasing the two primary characteristics of output during this period. Spain witnessed an initial recession in 2009, followed by a modest recovery, and a subsequent downturn in 2012 and 2013, succeeded by another recovery. Panel (b) shows the trajectory of nonperforming loans, also directly sourced from the data. Leading up to the 2012 crisis, the proportion of nonperforming loans surged, peaking at 9% in 2013. Finally, the financial shock κ, selected by the particle filter using both the data and our model, demonstrates a consistent decline from its peak value of 0.53 in 2008 to 0.32 in 2015.

[25] As in the calibration, we use the linearly detrended cyclical component of tradable output. Public debt is initialized at zero, and initial private debt is adjusted to match the composition of total debt in the data.

[26] We assume that only tradable output and nonperforming private loans are observed with no error. This leaves three observable variables not perfectly fitted in $\boldsymbol{Y}_t$: public debt, private debt, and spreads. To match them, there are three stochastic variables in $\boldsymbol{S}_t$, namely, B_t, L_t, and κ_t. By setting the variance of all measurement errors to 1% of their sample variance, we compute the filtered path of these three stochastic variables that is consistent with the data.

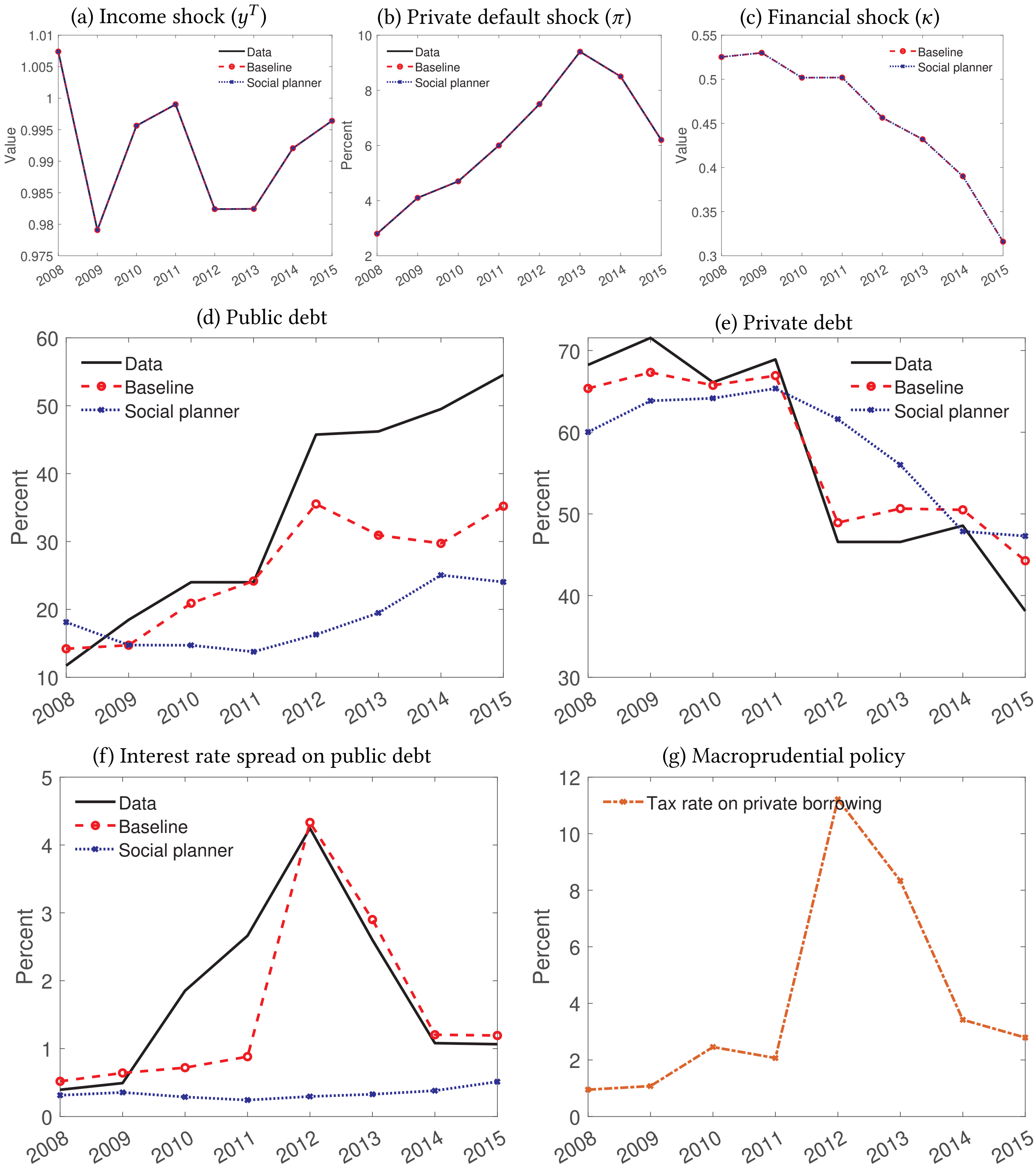

Figure 4: Evolution of debt, taxes, spreads, and exogenous shock, 2008–2015: data and models

Note: Model simulations are obtained by feeding into the model observed income shocks, nonperforming loans, and the most likely series of financial shocks from the particle filter. Public debt, private debt, and spreads are the particle-filtered weighted averages. Both debt series are expressed as a percentage of output, and nonperforming loans are expressed as a percentage of gross loans. Taxes and interest rate spreads are expressed in percentages. Data sources can be found in Appendix C, and details on the particle filter can be found in Appendix H.

This trend aligns with the narrative of external creditors shifting away from Spanish banks during this period. Given these income and financial shocks we will explain the endogenous responses of both the baseline and planned economy, in terms of private debt, public debt, and spreads.

Baseline model: The responses of the baseline model are illustrated as dashed red lines in Figure 4, effectively capturing the pivotal events of the 2012 crisis. Notably, the magnitude of the 2012 private debt deleveraging–approximately 12% of GDP—is mirrored by a corresponding surge in public debt. This coincides with an uptick in the interest rate spread on public bonds by approximately 4 percentage points, akin to the increase observed in the data. The model, however, is less successful at tracking the evolution of public debt after 2012, predicting a lower level of indebtedness compared to the data. Crucially, as in the data, the low output observed in 2009 doesn't coincide with an increase in spreads. This is because a decline in output leads to elevated spreads only when accompanied by high public debt. In 2009, the financial shocks weren't severe enough to induce deleveraging in private debt, thus averting bailouts and maintaining low public debt levels. Conversely, in 2012, the conjunction of low income and adverse financial shocks compelled the private sector to deleverage, prompting a public debt-financed bailout. In section **??** we explain why it is optimal for the government to provide large bailouts when confronted with substantial private deleveraging. Consequently, public debt experiences a sharp increase during a period of low default costs, precipitating a spike in spreads.

Planned economy: In contrast to the baseline scenario, the socially planned economy reacts to the same income and financial shocks with a gradual deleveraging in private debt and only a slight increase in public debt, primarily noticeable in 2014. As a result, interest rate spreads paid on public debt remain below 1% throughout the period. These allocations could have been implemented by the government through a macroprudential tax on private borrowing, as presented in panel (g). This analysis illustrates the counterfactual debt dynamics if the portfolio of private and public debt had been managed internalizing the two pecuniary externalities discussed in Section 3.5, and if the international lenders anticipated such management both before and after the crisis.

7 Policy functions

Default sets: In the baseline model the default decision is a function of both the exogenous states (s) and the endogenous states (B, L) of the economy. Panel (a) of Figure 5 shows the default sets of the government as a function of the income shock and the initial level of public debt (L). The model exhibits the standard result of the sovereign debt literature: the government defaults for low values of the income shock and high initial values of public debt. We plot the default sets for a low value of initial debt (blue) and a high value of initial private debt (orange). Low and high levels correspond to 22 and 42 percent of mean output respectively. In this example, we can see that on average the default sets expand when private debt increases. Intuitively, when households carry a higher private debt burden, the government is less inclined to impose additional taxes on them to repay the public debt as this will unambiguously tighten the borrowing constraint. Consequently, default becomes a more appealing option.

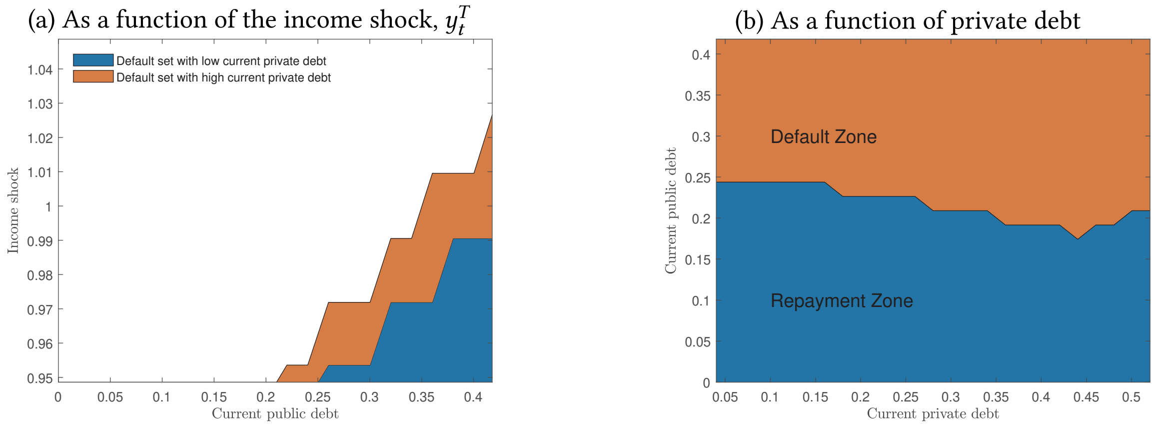

Figure 5: Default sets in the baseline model

Note: Default sets as a function of income and public debt (panel (a)), and as a function of public and private debt (panel (b)). A point is in the default set if the probability of default is above 50 percent. All debts are measured as a percent of output. In panel (a), the financial shocks are kept constant. High and low current private debt corresponds to 22 and 42 percent of mean output respectively. In panel(b), all financial shocks are kept constant and the income shock is three standard deviations below its mean.

In panel (b) of Figure 5, we investigate whether this outcome holds true across all potential levels of debt. Here, we maintain all exogenous shocks constant while plotting the default and repayment sets as a function of the initial levels of public and private debt. The income shock remains fixed at three standard deviations below its mean, and the financial shocks retain the low values from panel (a). As anticipated, we observe that, in general, default sets expand as the level of private debt increases. In this example, this trend persists until we reach a private debt level of approximately 45% of output. Beyond this high threshold of private debt, default sets begin to contract with the initial level of debt. As we will delve into later, this outcome is influenced by bailouts. When the initial level of private debt becomes exceedingly high, households face the prospect of a severe financial crisis without government assistance. Hence, ensuring continued access to international capital markets increases the government's incentives to repay.[27]

Private debt issuance: We now focus on the household's optimal issuance of private debt. This is a function of both the exogenous shocks, their own initial level of debt (B), and the government's transfer.[28]

To simplify the analysis we present the policy functions in a scenario where there is no initial public debt ($L = 0$). Panel (a) of Figure 6 shows the optimal private borrowing (b') as a function of the initial level of private debt (B), keeping fixed the exogenous shocks and the government transfer at zero ($L' = 0$ and therefore $T = 0$). For low levels of initial debt, the credit constraint does not bind, and end-of-period private debt monotonically increases with initial debt. The kink in the policy

[27]Note that the opportunity cost of accessing public bailouts serves as an endogenous default cost in this model. We credit and thank an anonymous referee for this observation and explore this channel in more detail in the next section.

[28]Since the optimal borrowing response as a function of the exogenous shocks is standard in this literature, we abstract from them in this analysis. For low levels of initial debt, end-of-period private debt decreases with income. However, if the current debt is high enough, households borrow up to their credit constraint, thus optimal borrowing becomes increasing in the endowment of tradables. Regarding the financial shocks, end-of-period debt increases with the borrowing capacity (increasing in κ and q).

function indicates the first point at which the credit constraint is satisfied with equality. Beyond this threshold, higher initial debt levels lead to reduced tradable consumption, thereby lowering the price of nontradables (p^N) and further constricting the borrowing capacity of the economy, and therefore inducing further deleveraging. The literature calls this mechanism the Fisherian debt deflation effect, first alluded in Fisher (1933). As a result, similar policy functions can be seen in all models where in equilibrium the value of collateral is increasing in consumption.

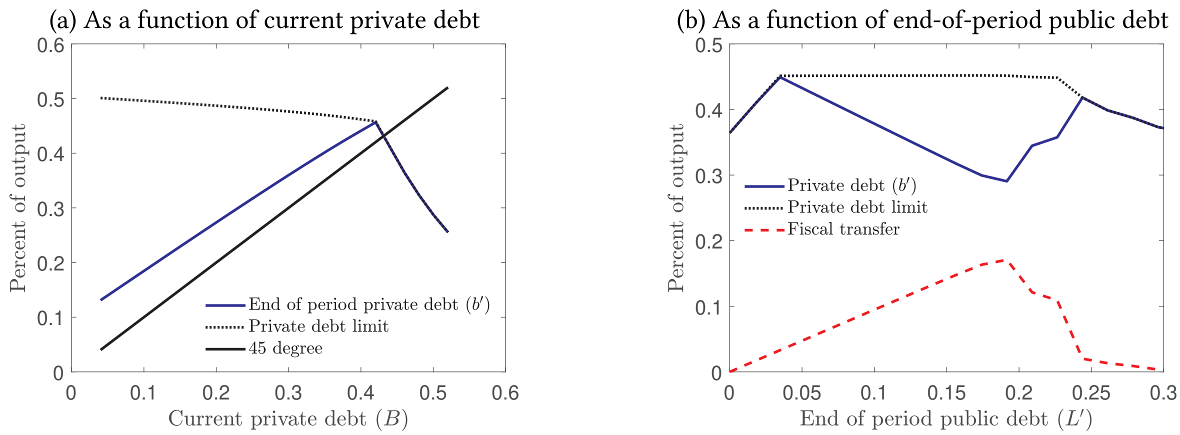

Figure 6: Policy function of private debt as a function of past debt and transfers

Note: Optimal private debt issuances (b') as a function of the initial level of private debt (panel (a)) and the end-of-period level of public debt (panel (b)). All debts are measured as percent of output. All financial shocks are kept constant and the income shock is three standard deviations below its mean. In panel (a), both initial and end-of-period public debt is zero ($L = L' = T = 0$). In panel (b), initial private debt is kept constant at 46 percent of mean output, and initial public debt is zero ($L = 0$).

In panel (b) of Figure 6, we analyze the impact of the government borrowing decisions on the household's private borrowing choices. To investigate this, we maintain constant exogenous shocks, set the initial level of public debt at zero ($L = 0$), and consider a very high initial level of private debt (B at 44 percent of mean output). In this depicted scenario, if the government maintains zero debt, the transfer is also zero, and the households will face a binding credit constraint.

A small issuance of public debt (less than 5 percent of output) implies a small positive transfer to households, which they allocate towards increasing their consumption of tradables. This surge in tradable consumption increases the price of nontradables and thus relaxes the private borrowing constraint. At these low transfer levels, households utilize all their newfound borrowing capacity, consuming more and further relaxing the constraint. Thus private borrowing increases as public debt (L') rises and transfers trigger a positive feedback that operates as Fisherian debt deflation in reverse.

Public debt choices ranging between 5 and 22 percent of output monotonically increase the value of the fiscal transfer. However, at these levels, the fiscal transfer becomes substantial enough to pull households away from their credit constraints. Consequently, additional public borrowing leads to equivalent reductions in private borrowing, ensuring that total tradable consumption remains relatively constant. This substitution between public and private debt occurs whenever households receive transfers and are unconstrained in their borrowing.

Finally, if the government decides to borrow more than 22 percent of output, its debt is issued at a high spread, causing the size of the fiscal transfer to decline. In response, households once again opt to increase their private borrowing. As the transfer dwindles towards zero, the private credit constraint becomes binding once more.

If the government starts with an initial level of debt to repay, the transfer is negative unless the issuances are large enough to cover repayment but not so large as to put the government on the wrong side of the Laffer curve.

Optimal public debt: The government takes into account the household's optimal responses when determining the level of public borrowing. Figure 7 illustrates the anticipated level of end-of-period public debt (blue line) as a function of the current level of private debt and the expected end-of-period private debt (red line) given government policies. Exogenous shocks are kept constant and initial public debt is held at its mean value at the ergodic state. Depending on the initial level of private debt, three distinct types of government responses regarding public debt are conceivable.

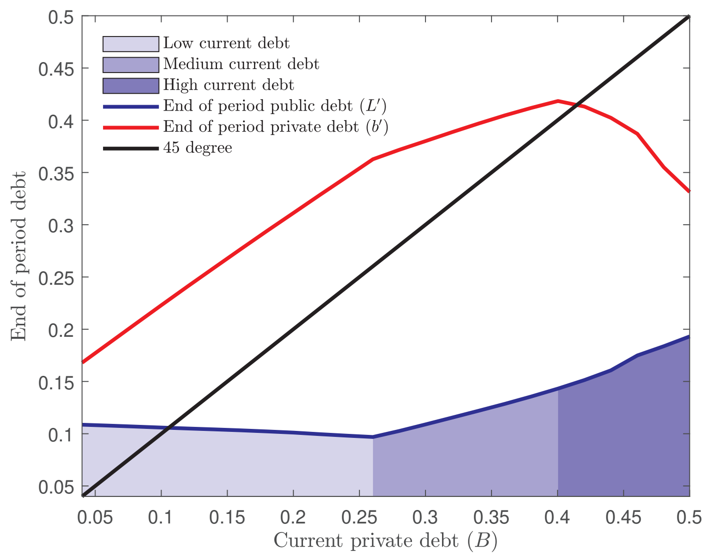

Figure 7: Expected end-of-period public and private debt as a function of initial debt

Note: Expected public debt issuance (L') and private debt issuances (b') as a function of the initial level of private debt (B). All debts are measured as percent of output. All financial shocks are kept constant and the income shock is three standard deviations below its mean. Current public debt (L) is kept constant at its mean value at ergodic distribution (15 percent of output).

When the initial level of private debt is low, the government seizes the opportunity to repay some of its debt. However, it does not repay the entire stock of public debt, as doing so would lead to increased private debt, and the government values the long-term bond's hedging benefits.[29] The econ-

[29] Compared to short-term bonds, long-term bonds offer a hedging advantage by reducing the debt burden to be repaid during unfavorable economic conditions. This finding also features prominently in the sovereign default literature with multiple maturities. As a result in both Arellano and Ramanarayanan (2012) and Hatchondo et al. (2016) a majority of total debt is issued in long-term bonds. However, these papers also indicate that short-term bonds become more prominent when default risk is high. Since our model does not incorporate endogenous default risk for short-term bonds, they tend to be the preferred choice for indebtedness on average.

omy is accumulating more private debt and reducing its public debt, indicating a substitution of public debt for private debt.

When initial private debt is at a medium level, the government foresees that without intervention, households may approach their borrowing limits. As a result, the government opts to borrow more and transfer those resources to households, which in turn moderates their pace of debt accumulation. Here, public debt substitutes for private debt.

Lastly, when initial private debt is high, absent government transfers the private credit constraint will bind and the households will have to deleverage substantially. Here, the government's optimal response is to increase even more its issuances of public debt. This response will relax the households' credit constraint and attenuate the need for extensive private deleveraging. Thus, public and private debts act as complements.

A comparison between the debt policy functions in this section and those of the social planner can be found in Appendix F.

8 Implications of the results

8.1 Implications of overborrowing for sovereign risk

Table 3 shows that private overborrowing increases sovereign risk. In this subsection, we argue that this is mostly driven by differences in the default rules of the planner and the government in the decentralized economy state-by-state. However, we also find, that in equilibrium, the economy is more frequently in states where higher private debt increases the probability of default.

In panel (a) of Figure 8 we return to exogenous states of Figure 5 and plot the repayment set of the baseline economy (orange) as a function of current private and public debt. We juxtapose this set with the repayment set of the planner at the same states (blue). The planner has indeed a strictly larger repayment set than the government on the baseline economy but the differences are small. Nevertheless, using the default rules of the planner but the borrowing choices of the baseline, we find that 44% of the defaults of the baseline start in a state in which the planner would have chosen to repay. Comparing this to the difference in default rates implies that 94% of the additional defaults in the baseline can be explained by the difference between the default sets.[30] The properties of the default set identified in the previous section for the baseline also hold for the planner. For most levels of debt, default sets expand with the initial level of private debt. The exception is for very large levels of private debt. However, at the ergodic distribution, the baseline and planner economy are only 0.7% and 0.4% of the time respectively in areas where more private debt reduces the default probability.

[30]To compute these statistics we use the 100000 simulations. We find 716 default episodes in the baseline and 385 for the planner. Thus, we have 331 excess defaults in the baseline relative to the planner. Among the 716 defaults in the baseline, 312 occur in a state where the planner would have chosen repayment. Conversely, all 385 defaults in the planner's equilibrium occur in states where the baseline government would also have chosen to default.

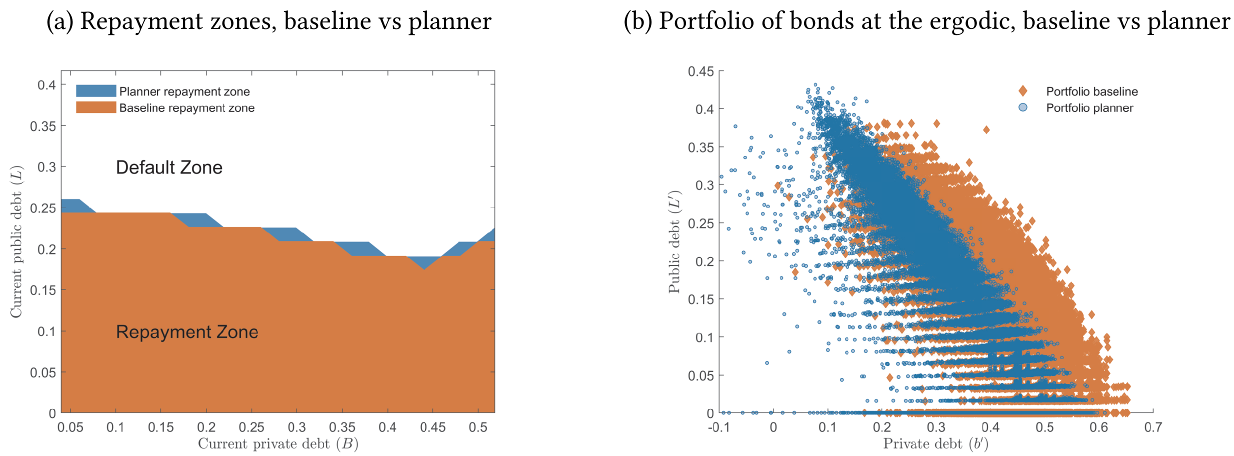

Figure 8: Changes in the repayment sets and portfolios at the ergodic

Repayment sets as a function of public and private debt for the baseline and planner economy (panel (a)) and portfolio of bonds at ergodic (panel (b)). A point is in the repayment set if the probability of default is below 50 percent. All debts are measured as a percent of output. In panel(a), all financial shocks are kept constant and the income shock is three standard deviations below its mean. Panel (b) is computed by simulating 100 500 periods of each model and cutting the first 500 periods out. We find 716 default episodes in the baseline and 385 for the planner. Among the 716 default episodes in the baseline, 312 start in a state in which the planner would have chosen repayment. Doing the symmetric exercise, we find no planner default episode where the government in the baseline would have chosen repayment.

In panel (b), we focus instead on the portfolio of debt of both economies at the ergodic distribution. We can see that the distribution of private debt is shifted to the right in the baseline economy relative to the planner (overborrowing) and that in both economies there is a strong negative correlation between the two types of debt. Thus, on average private and public debt behave more like substitutes: when private debt is high, public debt tends to be low, and vice versa. Sovereign risk will thus be high when private debt is falling.

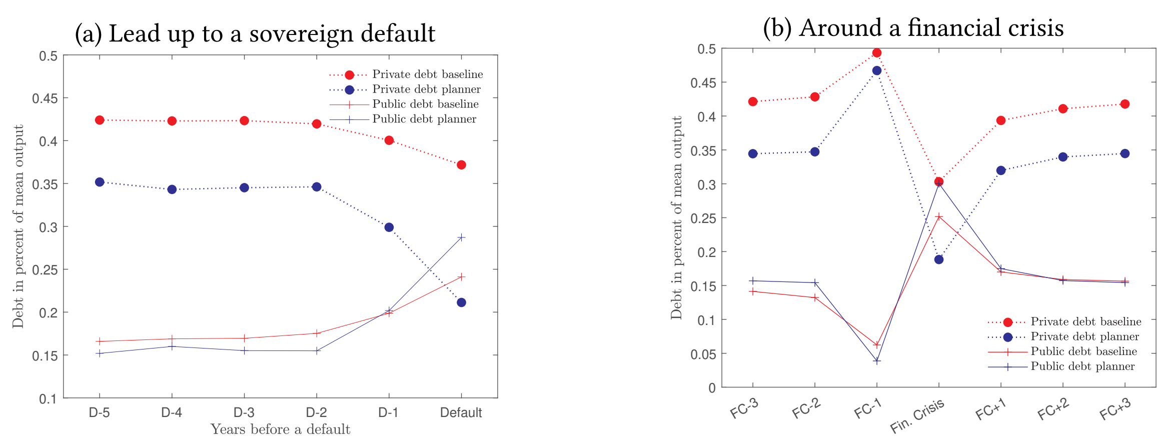

Figure 9: Portfolio dynamics during default and financial crises

Simulations of 100 500 periods of each model and cutting the first 500 periods out. We define a sovereign default as the first period in which the government defaults on the public debt. We define a financial crisis as a period in which the current account of private debt increases by more than two standard deviations above its mean. All debts are expressed as a percent of mean output at the ergodic distribution of the respective economy. We find 716 default episodes in the baseline and 385 for the planner. We also find 2507 financial crisis episodes in the baseline and 2073 for the planner.

We confirm this interaction between overborrowing and sovereign risk by showing the debt dynamics around sovereign defaults and financial crises in Figure 9. We do this for both the planner and the baseline economy. In panel (a), we show that sovereign defaults tend to follow episodes of rapid accumulation of public debt at the expense of deleveraging on private debt. The reduction in private debt in the lead-up to default, however, is larger for the planned than for the baseline economy. In panel (b), instead, we show how the portfolio evolves around a large contraction in the stock of private debt, i.e. a financial crisis. Here we recover the mirroring evolution of private and public debt positions that we observed for Spain in section 6. Note that at the peak of the crisis, the average level of public debt for both the planner and baseline is above the average level observed before a default, highlighting that default risk is also likely to be high during these crises.[31]

8.2 Implications for bailouts during crises

This section shows that bailouts are behind most of our welfare gains and that losing the ability to bail out the private sector is a quantitatively significant endogenous default cost. When the credit constraint binds, a positive fiscal transfer increases the consumption of tradables, and via this channel, it raises the relative price of nontradables, and therefore the borrowing capacity. To highlight the importance of this mechanism for our results we solve two alternative versions of the baseline model where this mechanism is shut down or very expensive to use.

Model with exogenous credit constraint In this model we substitute the credit constraint for private borrowing (2) with:

$$q_t b_{t+1} \leq \kappa_t (y_t^T + y_t^N)$$

This implies that the households' borrowing decisions will have no impact on either the current nor the future borrowing capacity via the relative price of nontradables ($\psi_t = \psi_{t+1} = 0$). Hence, in this model one of the two sources of overborrowing that we identify in section 3.5 is absent.[32] Moreover, this also means that bailouts will not have any impact on the borrowing capacity, and are therefore less effective than in the baseline economy. Columns three and four of Table 4 show the aggregate moments for this economy. We find that this specification exhibits much lower levels of overborrowing (2% of output as opposed to 8%). The welfare gains from internalizing the last remaining externalities are also close to zero, and the number for financial and default crises is roughly the same in the laissez-faire and planned economy.

Model with no exogenous default costs Another way of assessing the quantitative importance of bailouts in our baseline economy is to study an economy in which the exogenous default costs are zero ($\phi_0 = \phi_1 = 0$). In the standard sovereign default model, with no default penalty, the government

[31] Note that during crises, the economy substitutes private debt for public debt. Given our maturity assumption, the maturity of total debt thus increases during crises. This is at odds with the findings of the literature on sovereign default with multiple assets, and the data, but it is important to emphasize that in our model only the public bond is subject to endogenous default risk. Extending the model to allow for multiple types of defaultable public debt would bring the model in line with the data, but optimal maturity management is outside the scope of this paper.

[32] This would also be true in a one good model. Note that the second source of overborrowing, the fact that households don't internalize the effect of their private debt on the sovereign spread, is still present.

Table 4: Comparison to alternative models

			Exogenous credit constraint		No exogenous default costs	
	Baseline	Planner	Laissez-faire	Planner	Laissez-faire	Planner
Private debt as a % of output	43	35	31	29	46	44
Public debt as a % of output	16	15	15	15	3.9	4.1
Spread in percent	.81	.38	.49	.38	2e4	9e3
Prob. of a binding constraint	5.9	1.0	13	11	3.1	.80
Probability of a financial crisis	2.5	2.1	2.2	2.2	3.5	2.1
Probability of sovereign default	1.6	.90	1.2	1.2	45	44
Welfare gain	-	.26	-	.01	-	.06

Note: Simulated moments are computed at the calibrated parameters for different versions of the model. The first two columns correspond to the baseline and socially planned version calibrated in section 4. The third and fourth columns correspond to a version of the model where the credit constraint on private debt is purely exogenous ($\kappa(y^N + y^T)$). The third column corresponds to the decentralized case where competitive household choose their individual level of borrowing. The fourth column corresponds to the case where a benevolent social planner makes the aggregate borrowing decision. The fifth and sixth columns correspond to a version of the model with no exogenous default costs ($\phi_0 = \phi_1 = 0$). In the fifth column, households make the private borrowing decisions while in the sixth column, a benevolent social planner makes all aggregate borrowing decisions.

will default every period and the economy will not be able to sustain any positive level of public debt. In our setup, however, default also implies losing access to international credit markets and therefore the ability to bail out the private sector for a few periods. This acts in effect as an endogenous default cost. Columns five and six of Table 4 show that public borrowing is strictly positive in both the laissez-faire and planned versions of our model even in the absence of exogenous default costs and both economies exhibit default rates below 50 percent. Without exogenous default costs, however, spreads are high, and thus, welfare gains are less than a quarter of what we observe in the baseline.

8.3 Implication of sovereign risk for macroprudential policies

The presence of sovereign risk affects the optimal state-dependent tax on private debt that decentralizes the allocations that solve the socially planned problem (Proposition 1). To quantify the importance of this change, we also compute the optimal macroprudential tax for an economy with no sovereign debt. We first analyze the implications state by state and then at the ergodic. Figure 10 shows the expected optimal tax as a function of the initial level of private debt keeping initial public debt as well as the exogenous shocks at their mean values, for the two economies. The monotonicity of the optimal tax is the same as in Bianchi (2011); the tax increases as the economy approaches the collateral constraint and discontinuously drops to zero when the constraint binds. The presence of sovereign risk implies that the expected tax on borrowing is strictly positive for lower levels of initial debt relative to the economy with no public debt.

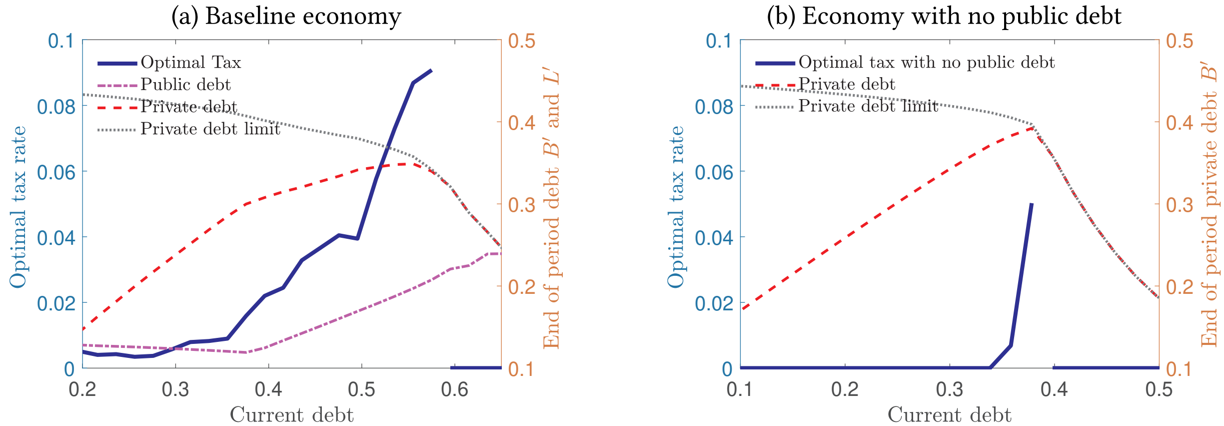

Figure 10: Optimal taxes as a function of total initial debt

Optimal taxes on private debt (τ) that decentralize the allocations of the planner at the expected levels of public borrowing (panel (a)) and in an economy without public debt (panel (b)). The exogenous shocks are the same for both plots with income shock set at three standard deviations below its mean. All debts are measured as percent of output. In panel (a), the initial level of public debt is set to its mean value at the ergodic (16%) while initial private debt varies along the x-axis from zero to fifty percent of output.

This difference is quantitatively important in equilibrium. Table 5 shows the aggregate moments and cyclicality of the optimal tax and Figure 11 shows the distribution of taxes observed at the ergodic distribution in a simulation of 10,000 periods. The average tax rate in an economy with sovereign risk is higher, 2.0%, as opposed to 1.6% in an economy with no public debt. Sovereign risk also makes taxes more volatile (2.6% vs. 1.6%). Moreover, the strong negative correlation between taxes and output in the economy with no public debt ($-.62$) is significantly attenuated in the baseline economy ($-.39$). The same is true for the correlation between the tax rate and private debt. All these changes stem from the fact that the optimal macroprudential tax is now also a function of the government's capacity to bail out the private sector which in turn is constrained by sovereign risk.

	Baseline	Economy with no public debt
Average tax rate	2.0	1.6
Volatility	2.6	1.6
Correlations		
Output - Tax rate	-.39	-.62
Total debt - Tax rate	.16	.28
Private debt - Tax rate	.13	.28
Public debt - Tax rate	-.06	-

Table 5: Moments at the ergodic

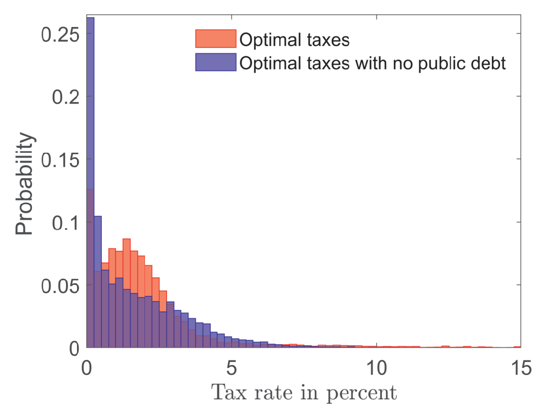

Figure 11: Density of optimal taxes

Optimal taxes on private debt at the ergodic distribution (panel (b)) in the baseline economy (red) and an economy with no public debt and hence no sovereign risk (blue), computed in a simulation of 10,000 observations. Panel (b) shows the first and second moments of the optimal tax as well as the correlations with output and with each type of debt.

9 Conclusion

We develop a quantitative theory of publicly and privately issued external debt featuring systemic externalities in private credit and sovereign risk. In our framework, decentralized households borrow above the socially optimal level because they don't internalize the aggregate externalities of their choices on their future borrowing capacity and on the price of government debt. The model is calibrated to Spain and is quantitatively consistent with both the near-zero interest rate spreads from 1999 to 2009, as well as with the subsequent surge observed in 2012.

During periods of favorable income and financial conditions, impatient households slowly accumulate a large stock of private debt. Throughout this phase, public debt remains low, and its price stays close to the risk-free rate. Eventually, a combination of adverse shocks materializes, and the private sector is forced to deleverage and reduce consumption. As aggregate consumption declines, the value of collateral also diminishes, exacerbating borrowing constraints. Observing this painful drop in aggregate consumption, the government responds by providing fiscal transfers financed through new issuances of public debt. Bailouts have a multiplicative positive effect, appreciating the value of collateral and expanding the private sector's borrowing capacity allowing the households to roll over more of their debt. Unfortunately, these gains come at a cost, more sovereign risk.

Relative to an economy where the aggregate externalities of private credit are internalized, our baseline model exhibits higher levels of private debt. Consequently, episodes of private deleveraging and publicly financed bailouts occur more frequently, resulting in higher average levels of public debt. Moreover, since bailouts are more likely during periods of low income, public debt is issued at higher spreads and default risk is higher.

We estimate the share of excessive private debt in Spain in the lead to 2012 to be approximately 8% of GDP. As a result, the annual probability of a sovereign default was 0.8 p.p. above the socially desirable level. Furthermore, we show that optimal borrowing policies could have been implemented by pairing public debt management with state-dependent taxes on private borrowing. We estimate an average tax rate of 2.0% for Spain, which is on average 0.4 p.p. higher than what a model without sovereign risk would suggest and exhibits almost double the volatility.

Several avenues for future research remain open. Exploring the quantitative implications of introducing moral hazard into the motivations for private overborrowing could yield fruitful insights. Additionally, investigating how budgetary covenants or fiscal limits could address incentives for bailouts and public debt dilution simultaneously, as seen in Aguiar and Amador (2018), presents an intriguing area for further exploration.

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Appendices

A Recursive competitive problem with taxes

For the representative household, the aggregate state of the economy includes the exogenous aggregate shocks denoted by $s = \{y^T, y^N, \kappa, \pi, \epsilon\}$, the initial level of government debt L, the initial level of aggregate private debt B, and the initial level of its own debt b. Following the same notation than in the body of the paper I denote $S = (s, L, B)$ the state space of the economy before government actions. Similarly, let $S_G = (S, d, L', \tau)$ denote the state space after government actions. Note that now that state includes the choice of taxes.

As before, households take as given the price of non-tradables $p^{N\tau}(S_G)$, the equilibrium price of price bonds $q^\tau(s)$, and government's current and future decisions regarding default $\boldsymbol{d}^\tau$, public debt $\mathcal{L}^\tau$, and taxes $\boldsymbol{\tau}$. They also know the functions associates with these choices, the lump-sum transfer $\mathcal{T}^\tau$. Finally, they also have a perceived law of motion of aggregate private debt $\mathcal{B}'^\tau$. The household's optimization problem in recursive form is:

$$V^\tau(S_G, b) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + \beta \mathbb{E}_s[V^\tau(S'_G, b')] \tag{22}$$

$$\text{subject to}$$

$$c^T + p^{N,\tau}(S_G)c^N + (1-\pi)b = y^T + p^{N,\tau}(S_G)y^N + q^\tau(s)(1-\tau)b' + T,$$

$$q^\tau(s)b' \leq \kappa[p^{N,\tau}(S_G)y^N + y^T],$$

$$(T, B', L', \tau) = \Big(\mathcal{T}^\tau(S_G), \mathcal{B}'^\tau(S_G), \mathcal{L}'^\tau(S_G), \boldsymbol{\tau}(S_G)\Big)$$

$$\text{And } S'_G = (s', L', B', \boldsymbol{d}^\tau(s', L', B'), \mathcal{L}^{\tau\prime}(s', L', B'), \boldsymbol{\tau}(s', L', B')).$$

Using the same notation than in the baseline case for the aggregate laws of motion of the private sector are $\mathcal{B}'^\tau(S_G)$, and $\{C^{i,\tau}(S_G)\}_{i=T,N}$, and public bond pricing $\boldsymbol{Q}^\tau(s, L', B')$ function. The government's problem is:

$$W^\tau(S) = \max_{d\in\{0,1\}} [1-d]W^{R,\tau}(S) + dW^{D,\tau}(S) \tag{23}$$

In case of default, $S_G = (S, 1, 0, \tau)$ and $W^{D,\tau}(S)$ is given by:

$$W^{D,\tau}(S) = \max_{\tau} u\Big(C^T, C^N\Big) + \epsilon^{Def} - \phi(y^T) + \beta \mathbb{E}_s \Big[\theta W^{\tau}\big(s', 0, B'(S_G)\big) + (1-\theta) W^{D,\tau}\big(S'\big)\Big] \tag{24}$$

subject to

$$\begin{aligned}
C^{T,\tau}(S_G) + (1-\pi)B &= y^T + q^{\tau}(s)(1-\tau)B' + T \\
C^{N,\tau}(S_G) &= y^N \\
T &= q^{\tau}(s)\tau B' \\
B' &= \mathcal{B}^{\tau\prime}(S_G) \\
S' &= (s', \mathcal{B}^{\tau\prime}(S_G))
\end{aligned}$$

Note that transfers can still be strictly positive in default since the government transfers the proceeds to of the private debt tax to the households. In case of repayment, $S_G = (S, 0, L', \tau)$ and the value is:

$$W^{R,\tau}(S) = \max_{\tau, L' \in \Lambda} u\big(C^{T,\tau}, C^{N,\tau}\big) + \epsilon(L') + \beta \mathbb{E}_s [W^{\tau}(s', L', B')] \tag{25}$$

subject to

$$\begin{aligned}
C^{\mathcal{T},\tau}(S_G) + (1-\pi)B &= y^T + q^{\tau}(s)(1-\tau)B' + T, \\
C^{\mathcal{N},\tau}(S_G) &= y^N, \\
T &= Q^{\tau}(s, L', \tau, B')[L' - (1-\delta)L] - \delta L + q^{\tau}(s)\tau B', \\
B' &= \mathcal{B}^{\tau\prime}(S_G)
\end{aligned}$$

The solution to the government's problem yields decision rules for default $\boldsymbol{d}^{\tau}(S)$, public borrowing $\mathcal{L}'^{\tau}(S)$, and taxes $\boldsymbol{\tau}(S)$. The transfers $\mathcal{T}^{\tau}(S_G)$ and preference shifter $D^{\tau}(S_G)$ are also pinned down by these decisions. The solution to the problem of competitive risk neutral foreign lenders yields the bond price schedule for private debt:

$$q^{\tau}(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r}, \tag{26}$$

and for public debt:

$$Q^{\tau}(s, L', B') = \frac{1}{1+r} \times \mathbb{E}_s \left[\left[1 - d' \right] \times \left[\delta + (1-\delta) Q^{\tau}\left(s', L'', B''\right) \right] \right], \tag{27}$$

$$\text{Where:}$$

$$B'' = \mathcal{B}^{\tau\prime}(s', L', B'),$$

$$L'' = \mathcal{L}^{\tau\prime}(s', L', B'),$$

$$d' = \boldsymbol{d}^{\tau}(s', L', B')$$

Definition 3. *A Markov regulated competitive equilibrium with taxes is defined by, a set of value functions $\{V^{\tau}, W^{\tau}, W^{R,\tau}, W^{D,\tau}\}$, policy functions for the private sector $\{\hat{b}^{\tau\prime}, \hat{c}^{T,\tau}, \hat{c}^{N,\tau}\}$, policy functions for the public sector $\{\boldsymbol{d}^{\tau}, \mathcal{L}^{\tau\prime}, \boldsymbol{\tau}\}$, a pricing function for nontradable goods $p^{N,\tau}$, pricing functions for public debt Q^{τ} and private debt q^{τ}, and perceived laws of motion $\{\mathcal{B}^{\tau\prime}, C^{T,\tau}, C^{N,\tau}\}$ such that*

1. *Given prices $\{p^{N,\tau}, q^{\tau}\}$, government policies $\{\boldsymbol{d}^{\tau}, \mathcal{L}^{\tau\prime}, \boldsymbol{\tau}\}$, and perceived law of motion $\mathcal{B}^{\tau\prime}$, the private policy functions $\{\hat{b}^{\tau\prime}, \hat{c}^{T,\tau}, \hat{c}^{N,\tau}\}$ and value function V solve the household's problem (22)*
2. *Given bond prices $\{Q^{\tau}, q\}$ and aggregate laws of motion $\{\tilde{B}^{\tau\prime}, \tilde{C}^{T,\tau}, \tilde{C}^{N,\tau}\}$, the public policy functions $\{\boldsymbol{d}^{\tau}, \mathcal{L}^{\tau\prime}, \boldsymbol{\tau}\}$ and value functions W^{τ}, $W^{R,\tau}$, and $W^{D,\tau}$, solve the Bellman equations (23)–(25)*
3. *Households' rational expectations: perceived laws of motion are consistent with the actual laws of motion $\{\mathcal{B}'(S_G) = \hat{b}^{\tau\prime}(S_G, B), C^{T,\tau}(S) = \hat{c}^{T,\tau}(S_G, B), C^{N,\tau}(S_G) = \hat{c}^{N,\tau} S_G, B)\}$*
4. *The private bond price function $q^{\tau}(s)$ satisfies (26)*
5. *Given public $\{\boldsymbol{d}^{\tau}, \mathcal{L}^{\tau\prime}, \boldsymbol{\tau}\}$, and private $\{\mathcal{B}^{\tau\prime}\}$, policies the public bond price $Q^{\tau}(s, \mathcal{L}^{\tau}(S)', \mathcal{B}^{\tau}(S_G)')$ satisfies (27)*
6. *Goods market clear:*

$$C^{N,\tau}(S_G) = y^N$$

$$C^{T,\tau}(S_G) + (1-\pi)B = y^T + q^{\tau}(s)\mathcal{B}^{\tau\prime}(S_G) + \left\{1 - \boldsymbol{d}^{\tau}(S)\right\} \times \tag{28}$$

$$\left\{ Q^{\tau}(s, \mathcal{L}^{\tau}(S)', \mathcal{B}^{\tau}(S_G)') \left[\mathcal{L}^{\tau\prime}(S) - (1-\delta)L \right] - \delta L \right\}$$

Similarly to the baseline model the optimality conditions of the households problem are:

$$q^{\tau}(s)(1 - \boldsymbol{\tau}(S) u_T(C^{T,\tau}(S_G)) = \beta \mathbb{E}_s[(1-\pi') u_T(C^{T,\tau\prime}(S_G))] + \mu^{\tau}(S_G) q^{\tau}(s),$$

$$p^{N,\tau}(S_G) = \frac{1-\omega}{\omega} \left(\frac{C^{T,\tau}(S_G)}{y^N} \right)^{\eta+1},$$

$$0 \leq \kappa(p^{N,\tau}(S_G)y^N + y^T) - q^\tau(s)\mathcal{B}^{\tau\prime}(S_G) \quad \text{with equality if } \mu^\tau(S_G) > 0,$$

where μ^τ is the Lagrange multiplier associated with the credit constraint.

B Proof of proposition 1

This is a proof by construction. We will show that the recursive equilibrium with taxes can be written as a government problem that coincides with the planning problem (15). Start from the recursive competitive equilibrium problem with taxes described in Appendix B.

The problem with taxes is equivalent to the recursive problem of a government given that chooses allocations for the current period while taking future policies and prices as given. Denote these policies $\{\boldsymbol{d}^\tau(S), \mathcal{L}^{\tau\prime}(S), \boldsymbol{\tau}(S), C^{T,\tau}(S_G), C^{N,\tau}(S_G), \mathcal{B}^{\tau\prime}(S_G)\}$. This government maximizes utility considering the optimal responses of households and lenders. This is equivalent to let the government choose all policies using the Kuhn-Tucker conditions of households and lenders as constraints. The problem is therefore:

$$W^\tau(S) = \max_{d\in\{0,1\}} [1-d]W^{R,\tau}(S) + dW^{D,\tau}(S),$$

Let $S' = (S', B', L')$ the default value $W^{D,\tau}(S)$ is:

$$\begin{aligned}
W^{D,\tau}(S) &= \max_{c^T,c^N,B',\tau,\mu} u(c^T, c^N) - \phi(y^T) + \epsilon_{Def} + \beta\mathbb{E}_s\Big[\theta W^\tau(S') + (1-\theta)W^{D,\tau}(S')\Big] \\
&\text{subject to} \\
c^T + B(1-\pi) &= y^T + q^\tau(s)B', \\
c^N &= y^N, \\
q^\tau(s)B' &\leq \kappa\Big(p^{N,\tau}c^N + y^T\Big), \\
q^\tau(s)(1-\tau)u_T(c^T, c^N) &= \beta E_s[(1-\pi')u_T(C^{T,\tau}, C^{N,\tau}(S', \boldsymbol{d}^\tau(S'), \mathcal{L}^{\tau\prime}(S'), \boldsymbol{\tau}(S'))] + \mu q^\tau(s) \\
p^{N,\tau} &= \frac{1-\omega}{\omega}\Big(\frac{c^T}{c^N}\Big)^{1+\eta} \\
(\kappa(p^{N,\tau}c^N + y^T) - q^\tau(s)B')\mu &= 0 \\
\mu &\geq 0 \\
q^\tau(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r}
\end{aligned}$$

The value under repayment $W^{R,\tau}(S)$ is:

$$W^{R,\tau}(S) = \max_{c^T,c^N,B',\tau,\mu,L'\in\Lambda} u\big(c^T, c^N\big) + \epsilon(L') + \beta\mathbb{E}_s[W^\tau(S')]$$

$$\text{subject to}$$

$$c^T + B(1-\pi) + \delta L = y^T + q^\tau(s)B + Q^\tau(s, L', B')[L' - (1-\delta)L],$$

$$q^\tau(s)B' \leq \kappa\left(p^{N,\tau}c^N + y^T\right),$$

$$q^\tau(s)(1-\tau)u_T(c^T, c^N) = \beta\mathbb{E}_s[(1-\pi')u_T(C^{T,\tau}, C^{N,\tau}(S', \boldsymbol{d}^\tau(S'), \mathcal{L}^{\tau\prime}(S'), \boldsymbol{\tau}(S'))] + \mu q^\tau(s)$$

$$p^{N,\tau} = \frac{1-\omega}{\omega}\big(\frac{c^T}{c^N}\big)^{1+\eta}$$

$$(\kappa(p^{N,\tau}c^N + y^T) - q^\tau(s)B')\mu = 0$$

$$\mu \geq 0$$

$$q^\tau(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r}$$

$$Q^\tau(s, L', B') = \frac{1}{1+r} \times \mathbb{E}_s\left[\left[1 - \boldsymbol{d}^\tau(S')\right] \times \left[\delta + (1-\delta)Q^\tau\Big(s', \mathcal{L}^{\tau\prime}(S'), \mathcal{B}^{\tau\prime}(S', \boldsymbol{d}^\tau(S'), \mathcal{L}^{\tau\prime}(S'), \boldsymbol{\tau}(S'))\Big)\right]\right]$$

Substituting in the resource constraint for non tradables, and the intratemporal conditions that problem can be simplified to:

$$W^\tau(S) = \max_{d\in\{0,1\}} [1-d]W^{R,\tau}(S) + dW^{D,\tau}(S), \tag{29}$$

where default value $W^{D,\tau}(S)$ is:

$$W^{D,\tau}\big(S\big) = \max_{c^T,B',\tau,\mu} u\big(c^T, y^N\big) - \phi(y^T) + \epsilon_{Def} + \beta\mathbb{E}_s\Big[\theta W^\tau\big(S'\big) + (1-\theta)W^{D,\tau}\big(S'\big)\Big]$$

$$c^T + B(1-\pi) = y^T + q^\tau(s)B',$$

$$q^\tau(s)B' \leq \kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta}y^N + y^T\Big)$$

$$q^\tau(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r}$$

$$q^\tau(s)(1-\tau)u_T(c^T, y^N) = \beta\mathbb{E}_s[(1-\pi')u_T(C^{T,\tau}, C^{N,\tau})] + \mu q^\tau(s)$$

$$0 = \left[\kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta}y^N + y^T\Big) - q^\tau(s)B'\right]\mu$$

$$\mu \geq 0$$

and value under repayment $W^{R,\tau}(S')$ is:

$$W^{R,\tau}(S') = \max_{c^T,B',\tau,\mu,L'\in\Lambda} u\big(c^T, y^N\big) + \epsilon(L') + \beta\mathbb{E}_s[W^{\tau}(S')]$$

$$c^T + B(1-\pi) + \delta L = y^T + q^{\tau}(s)B + Q^{\tau}(s, L', B')[L' - (1-\delta)L] \tag{30}$$

$$q^{\tau}(s)B' \leq \kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta}y^N + y^T\Big) \tag{31}$$

$$q^{\tau}(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r} \tag{32}$$

$$Q^{\tau}(s, L', B') = \frac{1}{1+r} \times \mathbb{E}_s\bigg[\Big[1 - \boldsymbol{d}^{\tau}\Big] \times \Big[\delta + (1-\delta)Q^{\tau}\Big(s', \mathcal{L}^{\tau\prime}, \mathcal{B}^{\tau\prime}\Big)\Big]\bigg] \tag{33}$$

$$q^{\tau}(s)(1-\tau)u_T(c^T, y^N) = \beta\mathbb{E}_s[(1-\pi')u_T(C^{T,\tau}, C^{N,\tau})] + \mu q^{\tau}(s) \tag{34}$$

$$0 = \bigg[\kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta}y^N + y^T\Big) - q^{\tau}(s)B'\bigg]\mu \tag{35}$$

$$\mu \geq 0 \tag{36}$$

In this formulation it is apparent that the social planner problem (15) is a relaxed version of problem (29). In problem (29) the government must satisfy three additional constraints (34)–(36) and has access to two additional instruments μ and τ. Crucially, both μ and τ only appear in problem (29) in constraints (34)–(36). As such, problem (15) will be equivalent to problem (29) if we can use the solutions of (15) to construct two functions $\mu(s, L, B)$ and $\tau(s, L, B)$ that satisfy (34)–(36).

Let $\{C^{SP,T}(s, L, B), C^{SP,N}(s, L, B), \mathcal{L}^{SP\prime}(s, L, B), \mathcal{B}^{SP\prime}(s, L, B), \boldsymbol{d}^{SP}(s, L, B), Q^{SP}, q^{SP}(s)\}$ be a solution of problem (15). Additionally let $\mu^{SP}(s, L, B) \geq 0$ be the multiplier on the collateral constraint of the planner problem (15). μ^{SP} corresponds to the shadow value of relaxing the collateral constraint from the planner's perspective. This multiplier is different from μ which corresponds to the shadow value of relaxing the collateral constraint for individual households, and is a variable chosen by the government in (29). The complementary slackness condition of the social planner problem (15) is:

$$0 = \bigg[\kappa\Big(\frac{1-\omega}{\omega}\big(\frac{C^{SP,T}(s, L, B)}{y^N}\big)^{1+\eta}y^N + y^T\Big) - q^{SP}(s)\mathcal{B}^{SP\prime}(s, L, B),'\bigg]\mu^{SP}(s, L, B). \tag{37}$$

As such by setting:

$$\mu(s, B, L) = \mu^{SP}(s, L, B)$$

$$1 - \boldsymbol{\tau}(s, L, B) = \frac{\beta\mathbb{E}_s\Big[(1-\pi')\Big(u_T^{SP}(C^{SP,T}(S'), C^{SP,N}(S'))\Big)\Big] + \mu^{SP}(s, L, B)q^{SP}(s)}{q^{SP}(s)u_T(C^{SP,T}(s, L, B), y^N)},$$

We can see that (34)–(36) are satisfied and therefore the two problems are equivalent.

C Data Appendix

Gross Domestic Product (GDP): *Eurostat March 2019, National accounts aggregates by industry up to NACE A*64, nama_10_a64,-*. Corresponds to Total gross value added in all NACE activities. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2015.

Non-tradable share of GDP: *Eurostat March 2019, National accounts aggregates by industry up to NACE A*64, nama_10_a64.* Corresponds to the share of total value added produced in the following industries: public administration, wholesale and retail, construction, and real state. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2015.

Tradable share of GDP: *Eurostat March 2019, National accounts aggregates by industry up to NACE A*64, nama_10_a64.* Corresponds to the complement of nontradable valued added as a share of total value added. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2015.

Private debt: *Chapter 17 of the statistical bulletin of March 2019, Banco de España (2019), table 21c "Breakdown by institutional sector".* Corresponds to the inverse of the net international investment position of Spanish monetary financial institutions (excluding the Bank of Spain) and other resident sectors. The data series used are 3273771 and 3273777. Data is annualized from quarterly data from March 1999 to December 2015 and is in millions of Euros. In the calibration we use data only from 1999 to 2011,

Public debt: *Chapter 17 of the statistical bulletin of March 2019, Banco de España (2019), table 21c "Breakdown by institutional sector".* Corresponds to the inverse of the net international investment position of the Bank of Spain and all public administrations. The data series used are 2386960 and 3273774. Data is annualized from quarterly data from March 1999 to December 2015 and is in millions of Euros. In the calibration we use data only from 1999 to 2011,

Total debt: *Chapter 17 of the statistical bulletin of March 2019, Banco de España (2019), table 21c "Breakdown by institutional sector".* Corresponds to the inverse of the net international investment position of Spain and is calculated as the consolidation of private and public positions. Data is annualized from quarterly data from March 1999 to December 2015 and is in millions of Euros. In the calibration we use data only from 1999 to 2011.

Risk free rate: *Bloomberg ticker GTDEM1Y Govt*, Corresponds to the average interest rate spread paid on 1 year German treasury bonds. Data is annualized from quarterly data from March 1999 to December 2011.

Spread on public bonds: *Bloomberg tickers GTESP6YR Govt and GTDEM6Y Govt*, Corresponds to the difference between average interest rate paid on 6 year Spanish treasury bonds and 6 year German treasury bonds. Data is annualized from quarterly data from March 1999 to December 2015. In the calibration we use data only from 1999 to 2011.

Average Maturity: *Table 5 from the Bank of Spain's economic bulletin Alloza et al. (2019), of March 2019*, Average maturity of the stock of public debt for Spain in years. Annual data from 1999 to 2011.

Nonperforming loans: *Bloomberg ticker BLTLWESP Index*, Nonperforming loans as a share of total gross loans. Annual data from 1999 to 2015.

Consumption: *Eurostat , GDP and main components (output, expenditure and income) nama_10_gdp*. Corresponds to final consumption expenditure. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2017.

Current Account: *Eurostat, Balance of Payments BOP_GDP6_Q, table TIPSBP11*. Corresponds to current account as a percent of GDP. Definitions are based on the IMF's Sixth Balance of Payments Manual (BPM6). The data is unadjusted. Frequency is annual from 1999 to 2017.

Trade Balance: *Eurostat, Balance of Payments BOP_GDP6_Q, table TIPSBP11*. Corresponds to the balance of trade on goods and services as a percent of GDP. Definitions are based on the IMF's Sixth Balance of Payments Manual (BPM6). The data is unadjusted. Frequency is annual from 1999 to 2017.

D Solution Method: The Government's ex-ante problem

Following the approach of Dvorkin et al. (2021), I can re-write the government's Bellman equations before the ϵ shocks are realized. From an ex-ante point of view, the shocks ϵ make the default and borrowing decisions stochastic. By taking expectations over these shocks, the decisions can be viewed as probabilistic. If we view the previously defined equilibrium as a game between the private and public sector each period, the ϵ shocks allow the government to play mixed strategies. This makes the computation of this problem using value function iteration possible. We follow this approach to write (7) from a an ex-ante perspective. That is when all the aggregate states have realized except the ϵ. For this we summarize all other exogenous state variables in $z = (y^T, y^N, \kappa, \pi)$. As mentioned in the main text we assume that L' is a finite and bounded grid with $\mathcal{J}$ elements. Denote by $F(\epsilon)$ the joint cumulative density function of the taste shocks and by $f(\epsilon)$ its joint density function. To simplify notation in what follows, the following operator to denotes the expectation of any function

$Z(\boldsymbol{\epsilon})$ with respect to all the elements of ,

$$\boldsymbol{Z} = \mathbb{E}_{\epsilon} Z(\boldsymbol{\epsilon}) = \int_{\epsilon_1} \int_{\epsilon_2} \dots \int_{\epsilon_{\mathcal{J}+1}} Z(\epsilon_1, .., \epsilon_{\mathcal{J}+1}) f(\epsilon_1, .., \epsilon_{\mathcal{J}+1}) d\epsilon_1, ..d\epsilon_{\mathcal{J}+1} \tag{38}$$

Given this notation we have that:

$$\boldsymbol{W}(z, L, B) = E_{\boldsymbol{\epsilon}}[W(s, L, B)]$$

$$\boldsymbol{W}(z, L, B) = E_{\boldsymbol{\epsilon}}\Big[\max\big\{W^R(s, L, B); W^D(s, B)\big\}\Big]$$

$$\boldsymbol{W}(z, L, B) = E_{\boldsymbol{\epsilon}}\Big[\max\big\{\max_{L' \in \Lambda}\{u(C(s, L, B)) + \epsilon(L') + \beta \mathbb{E}_{z'|z} \boldsymbol{W}(z', L', \mathcal{B}'(s, L, B))\};$$

$$u(C(s, 0, B)) - \phi(y^T) + \epsilon^{Def} + \beta \mathbb{E}_{z'|z} \boldsymbol{W}(z', 0, \mathcal{B}'(s, 0, B))\big\}\Big]$$

Subject to the resource constraints:

$$C^T(s, L, B) = y^T + q(s)\mathcal{B}'(s, L, B) - (1 - \pi)B + Q(s, L', B')[L' - (1 - \delta)\mathcal{B}'(s, L, B)] - \delta\mathcal{B}'(s, L, B)$$

$$C^N(s, L, B) = y^N$$

Furthermore, if its convenient to define the following expected utility objects:

$$\Upsilon_{L,L'}(z, B) = u(C(s, L, B)) + \beta \mathbb{E}_{z'|z} \boldsymbol{W}(z', L, \mathcal{B}'(s, L, B))$$

$$\Upsilon_{def}(z, B) = u(C(s, 0, B)) - \phi(y^T) + \beta \mathbb{E}_{z'|z} \boldsymbol{W}(z', 0, \mathcal{B}'(s, 0, B))$$

Lemma 2. *Suppose that the $\boldsymbol{\epsilon}$ shocks follow a multivariate generalized extreme value distribution with parameters $\{m, v, p\}$ and are i.i,d over time. Where v is the scale parameter and p is the shape parameter and is set to 1. m corresponds to the location parameter and is set to $-v\gamma$ where γ is the Euler constant. Suppose that public debt L is on a grid with $\mathcal{J}$ points. Then the ex-ante value function of the government's recursive problem can be re-written as*

$$\boldsymbol{W}(z, L, B) = \Upsilon_{def} + v \log\left[1 + \left(\sum_{L' \in \Lambda} \exp\left(-\frac{\Upsilon_{def} - \Upsilon_{L,L'}}{pv}\right)\right)^p\right] \tag{39}$$

Additionally given this distributional assumptions there are closed form solutions for the ex-ante probability of default and borrowing policy functions conditional on repayment.

Proof. Given our distributional assumptions

$$F(\boldsymbol{\epsilon}) = \exp\left[-\left(\sum_{j=1}^{\mathcal{J}} \exp\left(-\frac{\epsilon_j - m}{v}\right)\right) - \exp\left(-\frac{\epsilon_{\mathcal{J}+1} - m}{v}\right)\right] \tag{40}$$

For $j \in [\![0, \mathcal{J}+1]\!]$ we denote by $F_j(\boldsymbol{\epsilon}) = \frac{\partial F(\boldsymbol{\epsilon})}{\partial \epsilon_j}$, the marginal with respect to element j^{th} element of $\boldsymbol{\epsilon}$.

$$F_j(\boldsymbol{\epsilon}) = \begin{cases} \frac{1}{v} \exp\left[- \left(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\epsilon_j - m}{v}) - \exp(-\frac{\epsilon^{def} - m}{v}) \right) \right] & \exp(-\frac{\epsilon_j - m}{v}) \quad for\ j = 1..\mathcal{J} \\ \frac{1}{v} \exp\left[- \left(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\epsilon_j - m}{v}) - \exp(-\frac{\epsilon^{def} - m}{v}) \right) \right] & \exp(-\frac{\epsilon^{def} - m}{v}) \quad for\ j = \mathcal{J}+1 \end{cases}$$

Using this notation ant the dropping the states (z, B) from the previously defined $\Upsilon_{L,L'}(z, B)$ functions we can compute the ex-ante policy functions of the government in close form solutions. Let the probability of default be $\boldsymbol{d}(z, L, B) = \mathbb{E}_{\boldsymbol{\epsilon}} \boldsymbol{d}(z, L, B, \boldsymbol{\epsilon})$. Note that:

$$\begin{aligned} \boldsymbol{d}(z, L, B) &= \int_{-\infty}^{\infty} F_{\mathcal{J}+1}(\Upsilon_{def} + \epsilon^{def} - \Upsilon_1, ..., \Upsilon_{def} + \epsilon^{def} - \Upsilon_{def}) d\epsilon^{def} \qquad (41) \\ &= \int_{-\infty}^{\infty} \frac{1}{v} \exp\left[- \left(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} + \epsilon^{def} - \Upsilon_j - m}{v}) - \exp(-\frac{\epsilon^{def} - m}{v}) \right) \right] \exp(-\frac{\epsilon^{def} - m}{v}) d\epsilon^{def} \\ &= \int_{-\infty}^{\infty} \frac{1}{v} \exp\left[- \exp(-\frac{\epsilon^{def} - m}{v}) \left(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} - \Upsilon_j}{v}) + 1 \right) \right] \exp(-\frac{\epsilon^{def} - m}{v}) d\epsilon^{def} \end{aligned}$$

Define $\exp(\phi_{def}) = 1 + \sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} - \Upsilon_h}{v})$. We can use this to rewrite (41) as:

$$\begin{aligned} \boldsymbol{d}(z, L, B) &= \int_{-\infty}^{\infty} \frac{1}{v} \exp\left[- \exp(-\frac{\epsilon^{def} - m}{v}) \exp(\phi_{def}) \right] \exp(-\frac{\epsilon^{def} - m}{v}) d\epsilon^{def} \\ &= \frac{1}{v \exp(\phi_{def})} \underbrace{\int_{-\infty}^{\infty} \exp\left[- \exp(-\frac{\epsilon^{def} - m - v\phi_{def}}{v}) \right] \exp(-\frac{\epsilon^{def} - m - v\phi_{def}}{v}) d\epsilon^{def}}_{=v} \\ &= \frac{1}{1 + \left(\sum_{L' \in \Lambda} \exp\left(- \frac{\Upsilon_{def} - \Upsilon_{L,L'}}{v} \right) \right)} \qquad (42) \end{aligned}$$

Where the last equivalence uses the fact that the PDF of the generalized extreme distribution integrates to 1. Similarly, conditional on repayment, the random component $\boldsymbol{\epsilon}$ make the public borrowing decisions random from an ex-ante perspective. Given a set of current aggregate states relevant for the government, it is useful to introduce the probability of choosing an amount of public debt L' *conditional on not defaulting* as:

$$\boldsymbol{G}_{z,L,B}(L') = \mathbb{P}_{\boldsymbol{\epsilon}}\big(L' | d(z, L, B, \boldsymbol{\epsilon}) = 0\big)$$

Using the same notation as before we have that for the L' that is the j^{th} element of Λ:

$$\begin{aligned}
\boldsymbol{G}_{z,L,B}(L') =&\frac{1}{1-\boldsymbol{d}(z,L,B)}\int_{-\infty}^{\infty} F_j(\Upsilon_j+\epsilon^j-\Upsilon_1,...,\Upsilon_j+\epsilon^j-\Upsilon_{def})d\epsilon^j\\
=&\frac{1}{(1-\boldsymbol{d}(z,L,B))v}\times\\
&\int_{-\infty}^{\infty}\exp\left[-\exp(-\frac{\epsilon^j-m}{v})\left(\sum_{h=1}^{\mathcal{J}}\exp(-\frac{\Upsilon_j-\Upsilon_h}{v})+\exp(-\frac{\Upsilon_j-\Upsilon_{def}}{v})\right)\right]\exp(-\frac{\epsilon^j-m}{v})d\epsilon^j
\end{aligned}$$

Defining $\exp(\phi_j)=\exp(-\frac{\Upsilon_j-\Upsilon_{def}}{v})+\sum_{h=1}^{\mathcal{J}}\exp(-\frac{\Upsilon_j-\Upsilon_h}{v})$, we can simplify:

$$\begin{aligned}
\boldsymbol{G}_{z,L,B}(L') =&\frac{1}{(1-\boldsymbol{d}(z,L,B))v}\int_{-\infty}^{\infty}\exp\left[-\exp(-\frac{\epsilon^j-m}{v})\exp(\phi_j)\right]\exp(-\frac{\epsilon^j-m}{v})d\epsilon^j\\
=&\frac{1}{(1-\boldsymbol{d}(z,L,B))v\exp(\phi_j)}\underbrace{\int_{-\infty}^{\infty}\exp\left[-\exp(-\frac{\epsilon^j-m-v\phi_j}{v})\right]\exp(-\frac{\epsilon^j-m-v\phi_j}{v})d\epsilon^j}_{=v}\\
=&\frac{1}{(1-\boldsymbol{d}(z,L,B))\exp(\phi_j)}
\end{aligned}$$

Finally this can be further simplified to:

$$\begin{aligned}
\boldsymbol{G}_{z,L,B}(L') =&\frac{1}{(1-\boldsymbol{d}(z,L,B))}\times\frac{\exp(\Upsilon_j/v)}{\exp(\Upsilon_{def}/v)+\sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v})}\\
=&\frac{\exp(\Upsilon_{def}/v)+\sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v})}{\sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v})}\frac{\exp(\Upsilon_j/v)}{\exp(\Upsilon_{def}/v)+\sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v})}\\
=&\frac{1}{\sum_{H\in\Lambda}\exp\left(\frac{\Upsilon_{L,H}-\Upsilon_{L,L'}}{v}\right)}
\end{aligned} \tag{43}$$

Finally the value $\boldsymbol{W}(z, L, B)$ is given by:

$$\boldsymbol{W}(z, L, B) = \sum_{j=1}^{\mathcal{J}+1} \int_{-\infty}^{\infty} (\Upsilon_j + \epsilon_j) F_j(\Upsilon_j + \epsilon^j - \Upsilon_1, ..., \Upsilon_j + \epsilon^j - \Upsilon_{def}) d\epsilon^j$$

$$= \sum_{j=1}^{\mathcal{J}} \int_{-\infty}^{\infty} \frac{\Upsilon_j + \epsilon_j}{v} \times$$

$$\exp\Bigg[- \exp(-\frac{\epsilon^j - m}{v}) \Bigg(\sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_j - \Upsilon_h}{v}) + \exp(-\frac{\Upsilon_j - \Upsilon_{def}}{v}) \Bigg) \Bigg] \exp(-\frac{\epsilon^j - m}{v}) d\epsilon^j$$

$$+ \int_{-\infty}^{\infty} \frac{\Upsilon_{def} + \epsilon_{def}}{v} \times$$

$$\exp\Bigg[- \exp(-\frac{\epsilon^{def} - m}{v}) \Bigg(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} - \Upsilon_j}{v}) + 1 \Bigg) \Bigg] \exp(-\frac{\epsilon^{def} - m}{v}) d\epsilon^{def}$$

$$= \sum_{j=1}^{\mathcal{J}} \exp(-\phi_j) \times$$

$$\Bigg[\Upsilon_j + m + v\phi_j + \underbrace{\int_{-\infty}^{\infty} (\frac{\epsilon_j - m - v\phi_j}{v}) \exp\big[- \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) \big] \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) d\epsilon^j \Bigg]}_{=v\gamma}$$

$$+ \exp(-\phi_{def}) \times$$

$$\Bigg[\Upsilon_{def} + m + v\phi_{def} + \underbrace{\int_{-\infty}^{\infty} (\frac{\epsilon^{def} - m - v\phi_{def}}{v}) \exp\big[- \exp(-\frac{\epsilon^{def} - m - v\phi_{def}}{v}) \big] \exp(-\frac{\epsilon^{def} - m - v\phi_{def}}{v}) d\epsilon^{def} \Bigg]}_{=v\gamma}$$

Where in the last equivalence we have used the fact that for all j:

$$\Upsilon_j + m + v\phi_j = \frac{(\Upsilon_j + m + v\phi_j) \int_{-\infty}^{\infty} \exp\big[- \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) \big] \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) d\epsilon^j \Big]}{v}$$

The last step (underscored in the above equations) uses one of the integral properties of the Euler constant. We now use the fact we assumed the distribution of shocks to be mean zero, that is $m = -\gamma v$. Using the definition of ϕ_{def} one can see that:

$$\exp(-\phi_{def}) [\Upsilon_{def} + v\phi_{def}] = \frac{\Upsilon_{def} + v \log(1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v}))}{1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v})}$$

The value of the government is then given by:

$$
\begin{aligned}
\boldsymbol{W}(z,L,B) &= \sum_{j=1}^{\mathcal{J}} \exp(-\phi_j)[\Upsilon_j + v\phi_j] + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
\boldsymbol{W}(z,L,B) &= \sum_{j=1}^{\mathcal{J}} \frac{\Upsilon_j + v\log(\exp(-\frac{\Upsilon_j-\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(-\frac{\Upsilon_j-\Upsilon_h}{v}))}{\exp(-\frac{\Upsilon_j-\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(-\frac{\Upsilon_j-\Upsilon_h}{v})} + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
\boldsymbol{W}(z,L,B) &= \sum_{j=1}^{\mathcal{J}} \frac{\Upsilon_j - \frac{v\Upsilon_j}{v} + v\log(\exp(\frac{\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v}))}{\exp(-\frac{\Upsilon_j}{v})(\exp(\frac{\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v}))} + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
\boldsymbol{W}(z,L,B) &= \frac{v\log(\exp(\frac{\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v}))}{\exp(\frac{\Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v})} \sum_{j=1}^{\mathcal{J}} \exp(\frac{\Upsilon_j}{v}) + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
\boldsymbol{W}(z,L,B) &= \frac{\Upsilon_{def} + v\log(1 + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v}))}{1 + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v})} \sum_{j=1}^{\mathcal{J}} \exp(\frac{\Upsilon_j - \Upsilon_{def}}{v}) + \exp(-\phi_{def})[\Upsilon_{def} + v\phi_{def}] \\
\boldsymbol{W}(z,L,B) &= \left[\frac{\Upsilon_{def} + v\log(1 + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v}))}{1 + \sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v})}\right]\left[\sum_{j=1}^{\mathcal{J}} \exp(\frac{\Upsilon_j - \Upsilon_{def}}{v}) + 1\right] \\
\boldsymbol{W}(z,L,B) &= \Upsilon_{def} + v\log(1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v})) \qquad (44)
\end{aligned}
$$

To sum up the distributional assumptions allow us to obtain closed form solutions for the ex-ante value function (44), the policy functions for default (42), the public borrowing conditional on repayment (43), ∎

Note that the functions $\boldsymbol{G}_{z,L,B}(L')$ and $\boldsymbol{d}(z,L,B)$ are sufficient to express all government decisions. Using the fact that the shocks are i.i.d over time, and assuming a guess $\boldsymbol{Q}$ of next price schedule functions, we can use $\boldsymbol{G}_{z,L,B}(L')$ and $\boldsymbol{d}(z,L,B)$ to write the pricing equation of public bonds (14):

$$Q(z,L',B') = q(z)\mathbb{E}_{z'|z}\Bigg[\big[1 - \boldsymbol{d}(z',L',B')\big]\big[\delta + (1-\delta)\sum_{L''\in\Lambda} \boldsymbol{Q}\big(z',L'',\mathcal{B}'(z',L',B')\big)\boldsymbol{G}_{z',L',B'}(L'')\big]\Bigg] \qquad (45)$$

In the quantitative section we assume that the shocks are mean zero ($m = -\gamma v$). We also assume that the shape parameter p is one, therefore taste shocks are independent from each other within the period as well. The scale parameter v is calibrated to match the variance of public debt in the data.

E Numerical Solution

In this section, we provide more detail about the solution methods we use to solve both the baseline and planner versions of the model described in the main text. For both solution methods we use the closed-form ex-ante solutions of the government's problem described in detail in Appendix D.

Baseline. This version is solved in three steps. The first step solves the household's problem while taking government policies and bond prices as given using time iteration method. The second step uses the implied policy functions of the private sector from the first step and the assumed bond schedules and computes the closed-form solutions that solve the government's ex-ante problem. Finally using private and public policy functions the schedule of private bonds is updated. Iterate until convergence in private en public policies.

- Construct a finite grid of initial public debt L and private debt B.
- Discretize the 3 exogenous shocks, income, financial shock and private default and its transition probability matrix using Tauchen approximation. Solve for the implied schedule of private bonds $q(\pi)$ using (13).
- Provide an initial guess of ex-ante policy functions for government default $\boldsymbol{d}(z, L, B)$, and borrowing probabilities conditional on repayment $\boldsymbol{G}(z, L, B, L')$.
- Provide an initial guess for the schedule of public bonds $\boldsymbol{Q}(z, L', B')$.
- Construct the implied transfer function $\boldsymbol{T}(z, B, L, L')$ using the government budget constraint (5).
- Taking all these functions as given find the optimal private borrowing $B'(z, L, B, L')$ and consumption decisions $C'(z, L, B, L')$ using the private sector Euler equation (20) to find the binding and non binding states.
- Given households optimal policies $B'(z, L, B, L')$, and $C'(z, L, B, L')$, and the guess schedule of public bonds $\boldsymbol{Q}(z, L', B')$, compute the ex-ante default and borrowing policy functions of the government using (42) and (43). Update the government policy functions.
- Compute the government ex-ante value function $W(z, L, B)$ using (44).
- Update the schedule of public bonds $\boldsymbol{Q}(z, L', B')$ using (45).
- Repeat until convergence in $W(z, L, B)$,$B'(z, L, B, L')$, and $C'(z, L, B, L')$, and $\boldsymbol{Q}(z, L', B')$ is achieved.

Social planner. This version is solved in three steps. The first step finds optimal private borrowing on a grid (*grid search method*) given an initial guess of public for each potential default and public borrowing decisions. The second step uses this optimal private borrowing policy and the assumed bond schedules to computes the closed form solutions for public borrowing and default and the value function. Finally using private and public borrowing policy functions the schedule of private bonds is updated. Iterate until convergence in private borrowing policies and the value function is achieved.

- Construct a finite grid of initial public debt L and private debt B.

- Discretize the 3 exogenous shocks, income, financial shock and private default and its transition probability matrix using Tauchen approximation. Solve for the implied schedule of private bonds $q(\pi)$ using (13).

- Construct a grid of potential private borrowing choices B'.

- Provide an initial guess of ex-ante policy functions for government default $\boldsymbol{d}^{SP}(z, L, B)$, and borrowing probabilities conditional on repayment $\boldsymbol{G}^{SP}(z, L, B, L')$.

- Provide an initial guess for the schedule of public bonds $\boldsymbol{Q}^{SP}(z, L', B')$.

- Taking all these functions as given find the optimal private borrowing $B^{SP\prime}(z, L, B, L')$ in the finite grid discarding all choices that violate the credit constraint (16) for each potential public borrowing and default decision.

- Given optimal private borrowing policy $B^{SP\prime}(z, L, B, L')$ and the guess schedule of public bonds $\boldsymbol{Q}^{SP}(z, L', B')$, compute the ex-ante default and borrowing policy functions of the planner using (42) and (43). Update the planner's public borrowing and default policy functions.

- Compute the ex-ante value function $W^{SP}(z, L, B)$ using (44).

- Update the schedule of public bonds $\boldsymbol{Q}^{SP}(z, L', B')$ using (45).

- Repeat until convergence in $W^{SP}(z, L, B)$,$B^{SP\prime}(z, L, B, L')$, and $\boldsymbol{Q}^{SP}(z, L', B')$ is achieved.

F Policy functions of the planned economy

Figure 12 compares the evolution of end-of-period private debt in the baseline and socially planned economy with respect to the initial stock of private debt (panel (a)) and end-of-period public debt (panel (b)). In both panels, overborrowing in the baseline economy occurs only when the constraint does not bind. When the constraint binds, private borrowing is pinned down by the resource constraints, leading to no divergence between the models. However, the sources of private overborrowing in both panels differ.

In the first panel, households overborrow for low levels of initial private debt because they do not fully internalize the marginal effect of their debt on the probability of facing a binding constraint in the next period. This pattern is typical in models of private overborrowing with a credit constraint that increases with the price of nontradables.

In the second panel, households once again reduce their private borrowing when the transfer is sufficiently large. However, the planner's reduction in private borrowing is more pronounced. Unlike the planner, households do not fully internalize that higher private debt increases the probability of sovereign default in the next period. Consequently, individual households substitute less private debt for the same increase in public debt relative to the planner.

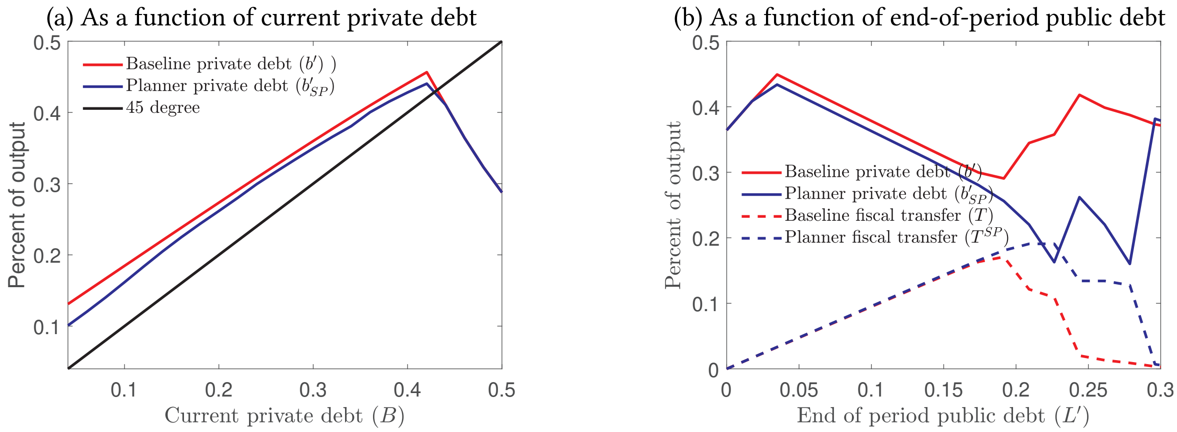

Figure 12: Policy function of private debt, baseline vs planner

Note: Optimal private debt issuances (b') as a function of the initial level of private debt (panel (a)) and the end-of-period level of public debt (panel (b)). All debts are measures as percent of output. All financial shocks are kept constant and the income shock is three standard deviations below its mean. In panel (a), both initial and end of period public debt is zero ($L = L' = T = 0$). In panel (b), initial private debt is kept constant at 46 percent of mean output.

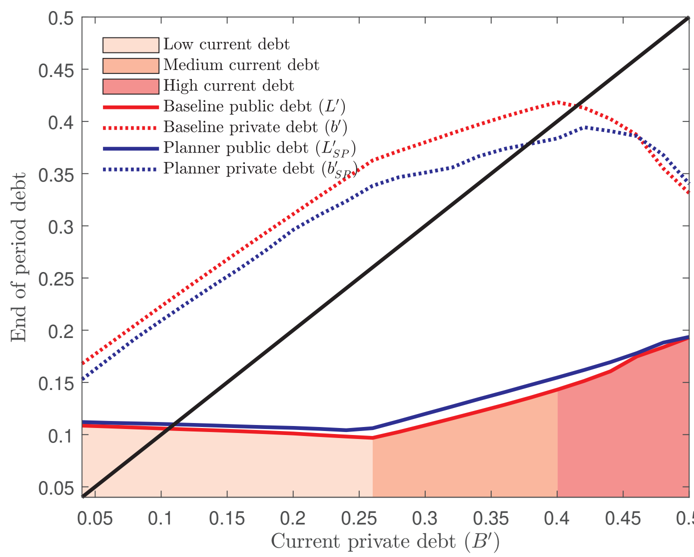

Figure 13: Expected end-of-period public and private debt as function of initial debt

Note: Expected public debt issuance (L') and private debt issuances (b') as a function of the initial level of private debt (B). All debts are measures as percent of output. All financial shocks are kept constant and the income shock is three standard deviations below its mean. Current public debt (L) is kept constant at its mean value at ergodic distribution (15 percent of output).

Figure 13 compares the expected optimal level of public borrowing, conditional on repayment, in the baseline and socially planned economies as a function of the initial debt. The responses of private debt by households are also plotted alongside those of the planners.

In the baseline model, private overborrowing occurs when the constraint does not bind. Conversely, public borrowing is higher in the planned economy everywhere, but the difference is larger when the constraint doesn't bind. We observe again the same three types of responses depending on the initial level of private debt. Note however that the low initial debt zone of the planner is slightly smaller. This means that the planner begins substituting private for public debt at lower levels of initial debt.

Although on a state-by-state basis, the planner is expected to issue more public debt than the baseline, at the ergodic distribution, the baseline model exhibits a higher average level of public debt. These disparities primarily stem from the baseline economy operating at a higher level of private debt, resulting in more public debt either to prevent a binding constraint (medium current debt) or to mitigate its consequences (high current debt). Thus, it is the frequency of bailouts that explains the extent of public overborrowing. When the constraint is expected to bind, the policies of the two economies mostly coincide. However, a slight amount of underborrowing in the baseline economy in this context is attributable to the planner facing a more favorable price schedule and thus being able to relax the constraint slightly more.

G Untargeted business cycle properties

This subsection evaluates the model's quantitative performance by comparing untargeted moments from the data with moments from the model at the ergodic distribution. I compute the model's moments by simulating the exogenous processes for 10,000 periods and eliminating the first 500 observations. The moments from the data are computed with annual data for the sample period 1999-2017. The longer sample period is chosen to avoid small sample bias. Similar results are obtained when restricting the sample to the period 1999-2011. In Table 6, real GDP is equated with output, and consumption corresponds to total final consumption expenditure and is measured in real terms. GDP and consumption data are detrended. The current account and trade balance are computed as a percentage of GDP. All data are from Eurostat, and additional descriptions of the sources can be found in Appendix C.

Table 6 compares the unconditional second moments in the Spanish data with their baseline model counterparts at the ergodic distribution. The model successfully captures the volatility of consumption, of the current account and of the trade balance, and overestimates the volatility of output. Nevertheless, the model correctly predicts that the volatility of output will exceed the volatility of consumption. This contrasts with traditional sovereign default models where the opposite is true.[33] This suggests that explicitly modelling international private debt is important to simultaneously achieve a volatility of consumption and net capital flows consistent with the Spanish data. Table 6 also computes correlations between output and the other business cycle statistics. The model correctly predicts the sign of all the correlations.

[33] Neumeyer and Perri (2005) find that consumption is more volatile than output in emerging economies whereas the opposite is true in advanced economies. Spain is listed by the International Monetary Fund (IMF) as an advanced economy.

Table 6: Untargeted business cycle statistics

Statistic	Data	Calibration
Volatility		
Output	.032	.044
Consumption	.031	.025
Current account	.041	.033
Trade balance	.034	.026
Correlations		
Output - Consumption	.97	.98
Output - Current account	-.59	-.82
Output - Trade balance	-.54	-.89
Output - Spread on public debt	-.46	-.09
Public debt - Spread on public debt	.53	.40

Note: Output corresponds to real gross domestic product and consumption to real final consumption expenditure, and both series are detrended. Current account and trade balance are measured as a percentage of output. Public debt corresponds to the international investment position of the public sector. Spreads correspond to the difference between the interest rate paid by Spanish six-year bonds and their German equivalents. For additional details, see Appendix C.

H Particle filter method

This appendix details the particle filter method used to conduct the counterfactual exercises of section 5. It follows closely the approach presented in Bocola and Dovis (2019). As noted in the main text, the state space representation of the model is:

$$\boldsymbol{Y}_t = g(\boldsymbol{S}_t) + e_t \tag{46}$$

$$\boldsymbol{S}_t = f(\boldsymbol{S}_{t-1}, \varepsilon_t). \tag{47}$$

In this formulation, the first equation captures the measurement error e_t, a vector of i.i.d. normally distributed errors with mean zero and a diagonal variance-covariance matrix Σ. The vector of observable, $\boldsymbol{Y}_t$, includes average private and public debt as share of GDP, detrended tradable output, the share of nonperforming loans, and interest rate spreads on public bonds. The second equation describes the law of motion of the baseline model state variables $\boldsymbol{S}_t = [L_t, B_t, y^T_{t-1}, \pi_{t-1}, \kappa_{t-1}]$. The vector ε_t corresponds to the innovations in the AR 1 process of the three structural shocks $[y^T_t, \pi_t, \kappa_t]$.

$$y^T_t = \exp(\rho^y \ln y^T_{t-1} + \varepsilon^y_t)$$
$$\pi^T_t = \exp((1-\rho^\pi)\bar{\pi} + \rho^\pi \ln \pi_{t-1} + \varepsilon^\pi_t)$$
$$\kappa_t = (1-\rho^\kappa)\bar{\kappa} + \rho^\kappa \kappa_t + \varepsilon^\kappa_t$$

Since we did not observe any defaults in the time periods considered we use the repayment policy functions to compute the transitions. Using the notation of section 3 the evolution of private and public debt in the first exercise is then:

$$L_{t+1} = \mathcal{L}'(s_t, L_t, B_t) = \mathcal{L}'(y_t^T, \pi_t, \kappa_t, 0, L_t, B_t)$$
$$B_{t+1} = \mathcal{B}'(s_t, L_t, B_t) = \mathcal{B}'(y_t^T, \pi_t, \kappa_t, 0, L_t, B_t)$$

In the first exercise all taste shocks are set to to zero. In the second exercise, we still focus on repayment but this time we select the taste shocks to match public debt exactly to it's data counter part and let private debt the respond endogenously:

$$L_{t+1} == L_{t+1}^{data}$$
$$B_{t+1} = \tilde{B}'(y_t^T, \pi_t, \kappa_t, L_t, B_t, 0, L_{t+1}^{data}, \tilde{T}(s_t, L_t, L_{t+1}^{data}))$$

These transitions are summarized in function $f(\cdot)$ for each exercise. Similarly we can generate numerical solutions to compute the model counterparts to debt to output ratios and the public spreads and summarize them in $g(\cdot)$.

Let $\boldsymbol{Y}^t = [\boldsymbol{Y}_1, ..\boldsymbol{Y}_t]$, and denote by $p(\boldsymbol{S}_t|\boldsymbol{Y}^t)$ the conditional distribution of the state vector given a history of observations up to period t. In general there is no analytical solution for the density function $p(\boldsymbol{S}_t|\boldsymbol{Y}^t)$. The particle filter method approaches this density by using the fact that the conditional density of $\boldsymbol{Y}_t$ given $\boldsymbol{S}_t$ is Gaussian. It consists of finding a set of pairs of states and weights $\{S_t^i, \tilde{w}_t^i\}_{i=1}^N$ such that for all function $h(\cdot)$:

$$\frac{1}{N}\sum_{i=1}^{N} h(\boldsymbol{S}_t^i)\tilde{w}_t^i \underset{a.s}{\longrightarrow} \mathbb{E}[h(\boldsymbol{S}_t)|\boldsymbol{Y}^t].$$

This approximation can then be used to obtain the weighted average path of the state vector over the sample. The states selected S_t^i are called particles and $\tilde{w}_t^i$ corresponds to their weight. To construct this set we follow the algorithm proposed by Kitagawa (1996).

Step 1: Initialization Set $t = 1$ and $\forall i \ \tilde{w}_0^i = 1$, draw S_0^i from the ergodic distribution of the baseline model.

Step 2: Transition For each $i = 1..N$ compute the state vector $\boldsymbol{S}_{t|t-1}^i$ given vector $\boldsymbol{S}_{t-1}^i$ by drawing innovations for the fundamental shocks from the calibrated distributions and using the policy functions summarized in $f(\cdot)$.

Step 3: Filter Assign to each particle $\boldsymbol{S}_{t|t-1}^i$ the weight

$$w_t^i = p(\boldsymbol{Y}|\boldsymbol{S}_{t|t-1}^i)\tilde{w}_{t-1}^i$$

where $p(\boldsymbol{Y}|\boldsymbol{S}^i_{t|t-1})$ is a multivariate Normal density.

Step 4: Rescale & Resample Rescale the weights $\{w^i_t\}$ so that they add up to one, and denote these new weights $\{\tilde{w}^i_t\}$. Sample with replacement N values of the state vector from the set $\{\boldsymbol{S}^i_{t|t-1}\}$ using $\{\tilde{w}^i_t\}$ as sample weights. Denote this draws $\{\boldsymbol{S}^i_t\}$. Set $\tilde{w}^i_t = 1\ \forall i$. If $t < T$ set $t = t + 1$ and go to Step 2. Otherwise, stop.

In both exercises, it is assumed that measurement error associated with y^T_t and π_t is zero, as such the variance of the measurement error is set to zero for these variables in the measurement equation and the innovations ε^y_t and ε^π_t are set to match the empirical counterparts exactly. Since κ_t has no empirical counterpart, the algorithm help us find the most likely path using its effects on debt aggregates and the spreads. As in Bocola and Dovis (2019) the filter is tuned with $N = 100,000$.

Equipped with a set of particles and weights $\{S^i_t, \tilde{w}^i_t\}^N_{i=1}$ and the policy functions summarized in $g(\cdot)$ one can approximate the model predictions plotted in figures 4. As an example for all $t = [2008, ..., 2015]$ the predicted interest rate spread, $spr^{Baseline}_t$ at time t is:

$$spr^{Baseline}_t = \sum^N_i \tilde{w}^i_t [\frac{\delta - \delta \boldsymbol{Q}(S^i_t)}{\boldsymbol{Q}(S^i_t)} - r]$$

Similar weighted averages are computed for the debt-to-output ratio and the exogenous shocks. When computing objects for the social planner the function $g^{SP}(\cdot)$ is used instead.

I Robustness: Alternative model specifications

I.1 Comparison to nested models

In this subsection, we conduct a comparative analysis of our model with two existing models of international borrowing, which are nested within our framework. This comparison offers insights into the distinct roles played by private and public debt in shaping the quantitative properties of our model. The findings of this comparison are summarized in Table 7. For consistency, we utilize the calibrated parameters outlined in section 4 to solve all models. Welfare gains are evaluated using the methodology outlined in section 5, expressed in terms of equivalent consumption.

First, we solve a model focusing solely on international private debt, subject to a collateral constraint. This mirrors the model presented in Bianchi (2011), with its characteristics outlined in the third and fourth columns of Table 7. Because of the pecuniary externality in private debt, two versions of this model exist a decentralized and a constrained efficient. Table 7 shows that both versions exhibit higher levels of private debt compared to our baseline model. However, the absence of public debt significantly improves the international debt position. It's notable that without a public debt, financial crises occur more frequently. This aligns with the result from section 8.2, the government

Table 7: Comparison to nested models from the literature

Related model			Bianchi (2011)	Bianchi (2011)	Hatchondo and Martinez (2009)	Arellano (2008)
Average	Baseline	Planner	Laissez-faire	Efficient	$\delta = .14$	$\delta = 1$
Private debt as a % of output	43	35	46	44	-	-
Public debt as a % of output	16	15	-	-	16	15
Spread in percent	.81	.38	-	-	.58	.13
Prob. of binding constraint	5.9	1.0	2.9	1.0	-	-
Probability of a financial crisis	2.5	2.1	3.7	2.8	-	-
Probability of sovereign default	1.6	.90	-	-	1.3	.28
Welfare gain relative to Baseline	-	.26	-2.0	-1.8	-1.2	-1.4

Note: All moments are obtained using 10,000 periods at the ergodic distribution. Simulated moments are computed at the calibrated parameters for different versions of the model. The first two columns correspond to the baseline and socially planned version calibrated in section 4. The third and fourth columns correspond to a version of the model with no public debt that coincides with the model presented in Bianchi (2011). The third column corresponds to the decentralized case where competitive household choose their individual level of borrowing. The fourth column corresponds to the case where a benevolent social planner makes the aggregate borrowing decision. The fifth and sixth columns correspond to a version of the model with no private debt. In the fifth column, the public debt is long-term and has the same maturity as in this paper. In the sixth column, the government only has access to one-period debt.

utilizes public debt to prevent crises by moving households further from their borrowing limits. Additionally, losing access to the long-term debt instrument heightens exposure to rollover risk. These two factors—-the increased frequency of crises and heightened rollover risk—-explain the welfare losses depicted in the final row.

Similarly, we compute a version of the model excluding private debt, corresponding to a two-goods variant of the standard Eaton and Gersovitz (1981) framework. The fifth column maintains the long-term maturity of the public bond, akin to Hatchondo and Martinez (2009). This results in a level of public debt close to the baseline model, yet with lower default probabilities and spreads, though still above those in the planned economy. Hence, it's evident that only when private debt is issued by competitive agents, as in the baseline, does sovereign risk escalate. Nevertheless, the loss of access to privately issued bonds also substantially reduces overall indebtedness, incurring costs for impatient households. Consequently, the overall effect on welfare is also negative. Finally, in the sixth column, maintaining the restriction on private borrowing while reducing the duration of public debt to one period, akin to Arellano (2008), yields similar outcomes to the preceding column. Debt levels are even lower, with defaults less frequent. Nonetheless, heightened exposure to rollover risk entails more significant welfare losses.

I.2 The role of long-term debt and the impact of the financial shocks

Duration: We solve the baseline model for different durations of public debt keeping all other parameters at their calibrated values. Columns three and four of Table 8 present the targeted moments

Table 8: Robustness of the moments for alternative parameters and shocks structure

Moments (in %)	Data	Baseline	$\delta/2$	2δ	$\kappa_t = \bar{\kappa}$	$\pi_t = \bar{\pi}$
Total debt	58	59	60	58	59	58
Private debt	43	43	43	42	43	42
Public debt	16	16	17	15	16	16
Mean spread	.73	.81	1.1	.58	.82	.81
Volatility debt	4.8	4.9	5.5	4.7	4.6	5.0
Volatility private debt	10	7.0	6.9	7.1	6.4	6.8
Volatility public debt	6.4	6.6	7.0	6.4	6.3	6.6
Volatility spread	.89	.73	.93	.71	.73	.80
Prob. of binding constraint	-	5.9	6.1	5.8	4.4	5.2
Prob. of a financial crisis	-	2.5	2.6	2.6	2.6	2.5
Prob. of default	-	1.6	2.1	1.2	1.6	1.9

Note: All moments are obtained using 10,000 periods at the ergodic distribution. The debt levels are expressed as percent of output. The interest rates, the probabilities, and interest rate spreads are in percent. Volatilities are standard deviations. A financial crisis is defined as a period in which the current account of the private sector increases by more than two standard deviations below the mean. Debt levels in the data are calculated using the international investment positions. More details are explained in Appendix C.

and implications for crises when the duration is 10 years ($\delta/2$) and 3 years (2δ) respectively.

Increasing the duration of public debt results in higher issuances of public debt but also higher spreads and more frequent defaults. Conversely, reducing the duration of the debt has the opposite effect, reducing reliance on public debt and lowering the default probability. With private debt being one period, the government can leverage the different spawning properties of the two types of debt if the duration of public debt is longer. However, this comes at the cost of higher welfare losses from debt dilution Aguiar and Amador (2014). This trade-off between dilution and rollover risk echoes findings in the sovereign default literature with multiple maturities Arellano and Ramanarayanan (2012).

Financial shocks (κ_t and π_t): In column five of Table 8, we present the moments of a version of the model in which the international financial shock κ_t is deactivated. Instead, we maintain κ_t constant at its mean calibrated value. In column six, we perform a similar analysis, but this time for the domestic financial shocks π_t, representing the share of private debt that is exogenously defaulted on. We then compare each of these moments to their counterparts in the data and the baseline model presented in columns one and two respectively.

The main takeaway from this table is that neither shock significantly changes the targeted moments, with two exceptions.[34] First, the volatility of private debt is notably lower in the absence of either shock compared to the baseline, and thus lower than its data counterpart. Therefore, in the baseline model, we include both of these shocks to better replicate the observed asymmetry in volatilities between private and public debt in the data. Second, without either shock, the probability of a binding credit constraint is lower than in the baseline, particularly for the κ shock. This underscores

[34] Models without financial shocks, such as Mendoza (2010), can still generate private crisis dynamics with realistic business cycle features.

the rationale for including these shocks. They introduce an extra source of variation to the borrowing capacity of private debt that does not directly affect the sovereign spread (unlike the income shock).

I.3 Alternative income structures

In this section, we explore the implications of deviating from the canonical sovereign default model, which typically features one final good that is perfectly traded. Instead, our model is a two-good economy with only income shocks affecting the tradable good. We compare our results to two alternative specifications that are closer to the canonical model.

First, we present a model with only tradable goods, which has only one source of overborrowing. However, this specification fails to generate financial episodes characterized by sudden and substantial deleveraging of private debt. As a result, it does not pass the validation exercise outlined in section 6.

Second, we consider a two-good economy where the nontradable endowment is stochastic and perfectly correlated with the tradable income shock ($y_t^N = y_t^T$). This specification yields aggregate results that closely resemble those obtained in our baseline model. Specifically, the average level of private overborrowing matches the measurement in our baseline framework. The aggregate moments of these two specifications are presented in Table 9.

Table 9: Comparison to models with different income shocks structures

			Only tradables model		Model with non tradable shocks y_t^N	
	Baseline	Planner	Laissez-faire	Planner	Laissez-faire	Planner
Total debt	59	50	56	55	58	50
Private debt	43	35	43	41	43	35
Public debt	16	15	13	13	16	15
Mean spread	.81	.38	.54	.49	.83	.39
Volatility debt	4.9	4.2	3.9	3.7	4.9	4.2
Volatility private debt	7.0	9.7	7.7	8.5	7.1	9.8
Volatility public debt	6.6	8.9	8.0	8.2	6.6	8.9
Volatility spread	.73	.38	.73	.52	.77	.39
Prob. of binding constraint	5.9	1.0	17	14	6.1	.94
Prob. of a financial crisis	2.5	2.1	2.8	2.1	2.7	2.1
Prob. of default	1.6	.90	1.1	.90	1.5	.93

Note: Simulated moments are computed at the calibrated parameters for different versions of the model. The first two columns correspond to the baseline and socially planned version calibrated in section 4. The third and fourth columns correspond to a version of the model with only tradable goods where the credit constraint is given by ($\kappa_t y_t^T$). The third column corresponds to the decentralized case where competitive household choose their individual level of borrowing. The fourth column corresponds to the case where a benevolent social planner makes the aggregate borrowing decision. The fifth and sixth columns correspond to a version of the model where the endowment of nontradable is stochastic and perfectly correlated with the tradable endowment shock ($y_t^N = y_t^T$). In the fifth column, households make the private borrowing decisions while in the sixth column, a benevolent social planner makes all aggregate borrowing decisions.

Model with only tradable goods: We consider a model with only tradables goods. This implies that composite consumption is equal to tradable consumption, and thus the flow utility is:

$$u(c_t^T) = \frac{(c_t^T)^{1-\sigma} - 1}{1-\sigma}$$

Since there are no longer relative prices the credit constraint is given by:

$$q_t b_{t+1} \leq \kappa_t y_t^T$$

As with the economy with an exogenous credit constraint presented in section 8.2, this implies that the households in the decentralized economy will fully internalize the effect of their borrowing policies on their current and future borrowing capacities. Thus one of the sources of overborrowing in our model is not present. Nevertheless, the second source of overborrowing is still present. The households do not internalize the effect of their private borrowing on current or future spreads ($Q(., L', B',)$) and on future government policies. As Table 9 shows, the impact for private debt is significant, the average level of overborrowing is only 2% of output[35] and the probability of binding credit constraints increases from 5.9% to 17% in the decentralized case.

Figure 14 presents the policy functions of private debt for the laissez-faire version of the one-good model in panel (a). This figure serves as a counterpart to Figure 7 in our baseline model. For low levels of private debt, the behavior of the one-good model resembles that of the baseline model. As the initial level of debt increases and the economy approaches its borrowing limit, public transfers rise. However, these transfers cannot be utilized to alleviate the credit constraint, unlike in the baseline model.

Furthermore, since the borrowing capacity remains fixed and does not endogenously decrease even with high initial debt levels, the economy does not undergo severe episodes of private debt deleveraging. Consequently, when attempting to conduct the same validation exercise outlined in section 6, this specification falls short. The deleveraging process is more gradual compared to the data, leading to public debt and spreads not reaching the levels observed in the data.[36]

Model with shocks to the endowment of nontradables (y_t^N) : An alternative way of approaching the canonical sovereign debt model is to allow for fluctuations in the endowment of nontradables. To simplify the computational burden of these two endogenous state models, we opt to allow for fluctuations in y_T^N that are perfectly correlated with the tradable income shock (y_t^T).[37] To preserve the

[35]Output in this economy is only the tradable endowment.

[36]It's worth noting that to perform this alternative validation exercise, we utilized the particle filter again to attempt to derive a plausible path of financial shocks κ_t to align this specification with the data.

[37]We thank the editor for suggesting this important counterfactual exercise.

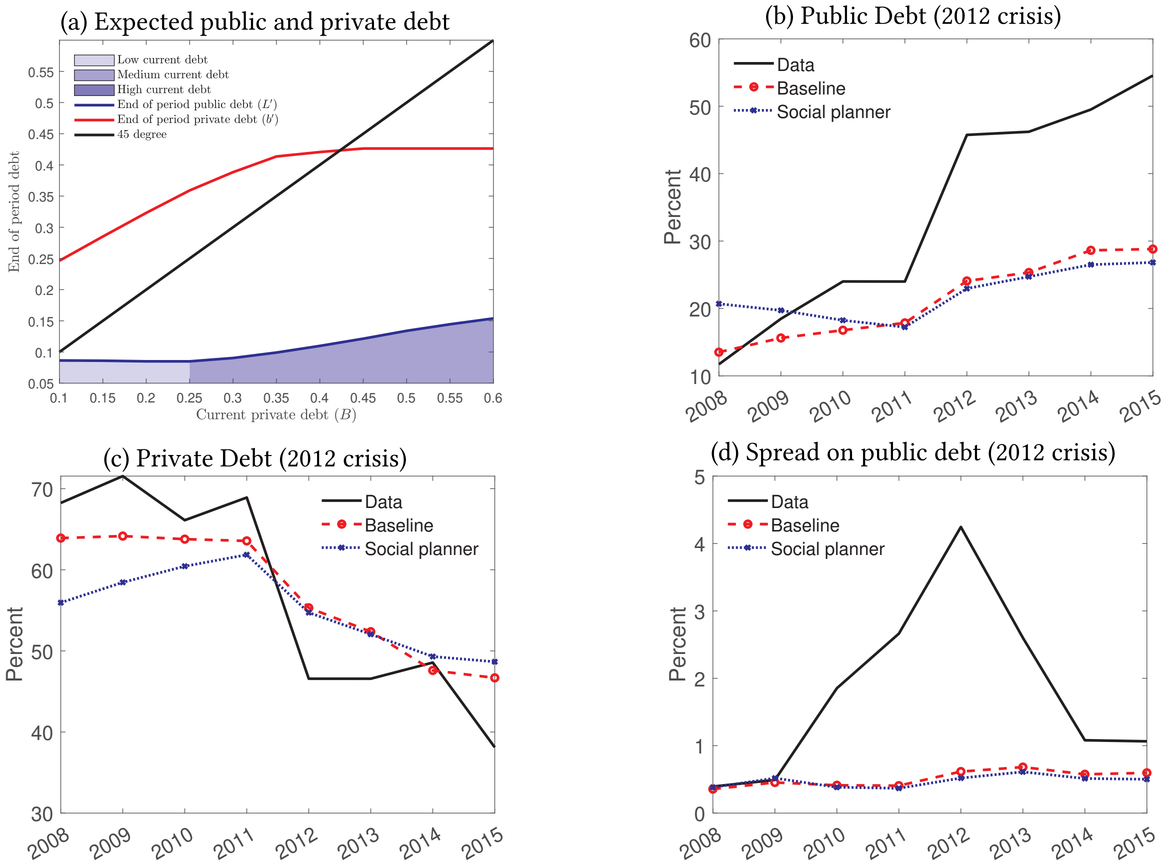

Figure 14: Only tradable goods model

Panel (a) is the expected public debt issuance (L') and private debt issuances (b') as a function of the initial level of private debt (B). All debts are measured as percent of output. All financial shocks are kept constant and the income shock is three standard deviations below its mean. Current public debt (L) is kept constant at its mean value at ergodic distribution (15 percent of output). Panel (b) to (d) are the results of the validation exercise in the model with shocks to nontradables. We feed the model the income and private default shock from the data around the 2012 crisis. We then use the particle filter to select the most plausible path for the financial shock κ_t. Panel (b) plots the evolution of public debt, panel (c) the evolution of private debt, and panel (d) the evolution of the interest rate spreads paid on public debt.

average level of y_t^N equal to one we assume that:[38]

$$y_t^N = y_t^T$$

Note that our two sources of overborrowing are still present in this specification of the model. However, the endogenous effect of tradable consumption on the borrowing capacity ($\psi = \kappa(1 + \eta)\frac{(1-\omega)}{\omega}\left(\frac{c^T}{y^N}\right)^\eta$) is attenuated by the perfect correlation assumption. This can also be seen in the planner's credit constraint:

$$q_t b_{t+1} \leq \kappa \left[\left(\frac{1-\omega}{\omega} \right) \frac{(c_t^T)^{1+\eta}}{(y_t^N)^\eta} + y_t^T \right]$$

Since the credit constraint is more likely to bind in periods when tradable income is low, the endogenous contraction of the borrowing capacity due to a reduction in c_t^T will be partly mitigated by

[38] Since the preference parameter ω is calibrated to match the observed share of tradables output in total output, a different average value of y^N would lead to a different calibration of ω which in turn, would keep the total income unchanged. A similar observation is made in footnote 9 of Bianchi et al. (2016)

the scarcity of the nontradable endowment that period (low y_t^N). However, our quantitative findings indicate that this mitigation effect is minimal and does not alter our primary results. It's worth noting that assuming a perfect correlation between y^N and y^T is a conservative assumption in our analysis. In a scenario where y^N and y^T are imperfectly correlated, low values of y^T might coincide with high values of y^N, potentially exacerbating the reduction in the relative price of nontradables and the deleveraging in private debt, thereby accentuating the issue of overborrowing even further.

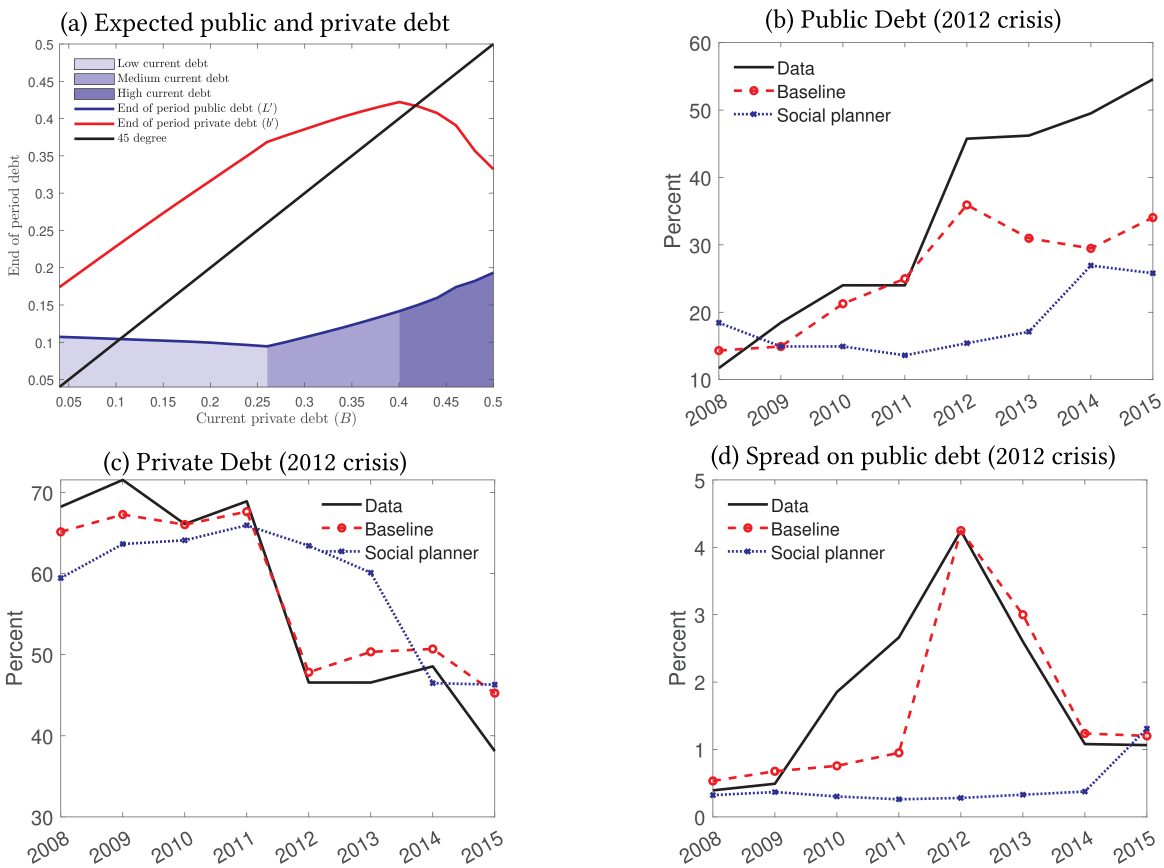

Figure 15: Model with shocks to the endowment of nontradables

Panel (a) is the expected public debt issuance (L') and private debt issuances (b') as a function of the initial level of private debt (B). All debts are measured as percent of output. All financial shocks are kept constant and the income shock is three standard deviations below its mean. Current public debt (L) is kept constant at its mean value at ergodic distribution (15 percent of output). Panel (b) to (d) are the results of the validation exercise in the model with shocks to nontradables. We feed the model the income and private default shock from the data around the 2012 crisis. We then use the particle filter to select the most plausible path for the financial shock κ_t. Panel (b) plots the evolution of public debt, panel (c) the evolution of private debt, and panel (d) the evolution of the interest rate spreads paid on public debt.

As shown in Table 9, the aggregate moments at the ergodic distribution of this specification are nearly identical to those in our baseline model. This holds true for both the laissez-faire and planned economies. Furthermore, as illustrated in Figure 15, the model with nontradable income shocks exhibits policy functions that closely resemble those in our baseline and also successfully passes the validation exercise of section 6. In the laissez-faire economy with nontradable shocks, spreads increased in 2012 due to a combination of low output, a drastic reduction in private debt, and high issuances of public debt. This reaffirms the result that output drops lead to significant spread increases only when coupled with high public debt. Additionally, spreads in the planned economy remain unchanged even during periods of low tradable output (e.g., in 2008, 2012, and 2013). Finally, permitting perfectly correlated shocks to nontradables also ensures that the untargeted business cycle statistics closely align with the values observed in the baseline model and the empirical data, as depicted in Table 10.

Table 10: Untargeted business cycle statistics baseline and extension

Statistic	Data	Baseline	Model with shocks to nontradables
Volatility			
Output	.032	.044	.043
Consumption	.031	.025	.030
Current account	.041	.033	.033
Trade balance	.034	.026	.026
Correlations			
Output - Consumption	.97	.98	.97
Output - Current account	-.59	-.82	-.84
Output - Trade balance	-.54	-.89	-.90
Output - Spread on public debt	-.46	-.09	-.08
Public debt - Spread on public debt	.53	.40	.38

Note: Output corresponds to real gross domestic product and consumption to real final consumption expenditure, and both series are detrended. The current account and trade balance are measured as a percentage of output. Public debt corresponds to the international investment position of the public sector. Spreads correspond to the difference between the interest rate paid by Spanish six-year bonds and their German equivalents. For additional details, see Appendix C.