Private Overborrowing under Sovereign Risk *

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Abstract

This paper argues that excessive international private debt increases the frequency and severity of sovereign debt crises. I develop a quantitative theory of private and public debt that allows me to measure the level of private overborrowing and its effect on the interest rate spread paid on public debt. In an environment where private credit is constrained by the market value of income, individually optimal private borrowing decisions are inefficient at the aggregate level. High private debt increases the probability of a financial crisis, characterized by a large deleveraging in private debt and a contraction in consumption. During such crises, drops in consumption cause a decline in the market value of collateral that in turn further reduces consumption. To counter this reduction, the government responds with large fiscal bailouts financed with risky external public debt. This response then causes a sovereign debt crisis, characterized by high interest rate spreads, and in some cases, default. I find that the theory is quantitatively consistent with the evolution of international private debt, international public debt, and sovereign spreads in Spain from 1999 to 2015. I estimate that private debt was 5% of GDP above the socially optimal level in the lead-up to the crisis. Private overborrowing increased the annual probability of a financial crisis by 2.4 percentage points. Finally, excessive private debt raised the interest rate spread on public bonds by at least 3.8 percentage points at its peak in 2012.

Keywords: Bailouts, credit frictions, financial crises, macroprudential policy, sovereign default

JEL Classifications: E32, E44, F41, G01, G28

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1 Introduction

A feature of the 2010-2015 European Debt Crisis is that governments that had previously pursued fiscally frugal policies saw significant increases in their borrowing costs. One of those countries was Spain. From the introduction of the euro in 1999 up to the global financial crisis in 2008, Spain was the largest economy in the Eurozone in uninterrupted compliance with the budgetary and public debt limits set by the Stability and Growth Pact.[1] During this same period, however, Spain accumulated a large stock of international private debt, primarily in its banking sector.[2] As the financial turmoil accelerated, the government responded with multiple rounds of bailouts to highly indebted financial institutions. This intervention led to an abrupt increase in public debt and its interest rate spreads. These events have raised questions about how private crises can lead to public debt crises and how a sovereign with defaultable debt should respond to systemic vulnerabilities in international private credit.[3] To address this issue, a joint analysis of private debt and sovereign risk is crucial in order to provide adequate policy prescriptions. Assuming a sovereign with full commitment could lead to policy prescriptions that are not sustainable. Conversely, assuming that bailouts must be financed only with funds raised within period would imply prescribing suboptimal policies that do not incorporate the gains from smoothing costs over time.

This paper sheds light on this problem by providing quantitative answers to the following three questions. First, was the Spanish private sector excessively indebted in the lead-up to the crisis and, if so, by how much? Second, by how much did private overborrowing increase the probability of a financial crisis? Third, what was the effect of excessive private debt on the severity of the sovereign debt crisis that followed?

I find that a model of systemic externalities in private credit regulated by a sovereign that can borrow internationally without commitment is quantitatively consistent with the evolution of private debt, public debt, and interest rate spreads in the Spanish data. The calibrated model matches the Spanish environment before the crisis – namely, low public debt, high private debt, and near-zero interest rate spreads. The calibrated model's untargeted business cycle statistics are also consistent with the data. I then verify that calibrated model can approach the crisis years, by computing the model dynamics using the productivity and external shocks taken from the Spanish data from 2008 to 2015. As in the data, the government in the model finds it optimal to provide large transfers to the

[1] Morris et al. (2006) discuss the reform of the Pact in 2005 and distinguish Spain for its compliance. Schuknecht et al. (2011) describe the evolution of deficits and sovereign debt in the post-reform period and document Spanish compliance up to the 2008 recession.

[2] Lane (2013) and Chen et al. (2013) discuss the current account imbalances of periphery European countries. Hale and Obstfeld (2016) and Hobza and Zeugner (2014) analyze capital flows within the Eurozone and document the flow in the form of debt instruments from "core" countries toward financial institutions in the periphery. In't Veld et al. (2014) and Ratto and Roegera (2015) link the increase in capital flows to Spanish banks financing a boom in the construction sector.

[3] This is not the first time that private credit booms have been linked to subsequent sovereign debt crises. An earlier literature analyzing the 1997 currency crises in Thailand, Korea, and Indonesia stresses this link. Burnside et al. (2001) argue that implicit bailout guarantees lead to private credit booms and rise expectations of large fiscal deficits in the future. Schneider and Tornell (2004) show that systemic bailout guarantees cause both credit cycles and self-fulfilling crises.

private sector financed with external public debt. This response in turn leads to a sudden decrease in private debt and a rise in the public interest rate spread commensurate with the increase observed in Spain. As a result, the paper contributes by providing a theory that jointly rationalizes the period of near-zero spreads and the high values observed during the crisis years.

This paper also provides quantitative estimates of the size of private overborrowing and its effects. I measure the excessive private debt stock from 1999 to 2011 to be 5% of GDP on average. I then define a financial crisis as a contraction of more than one standard deviation below the mean of the current account of the private sector. Under this definition, I estimate that private overborrowing increased the probability of a crisis in Spain from 0.1% to 2.5%. I also construct counterfactual dynamics of the model around the crisis years if private borrowing had been socially optimal. I show that even when taking public policies as given, the interest spread paid on public debt would have been 380 percentage points (p.p.) below the peak observed in 2012.

In addition to the main findings, this paper also delivers secondary findings. I find that private overborrowing increased the annual probability of observing a sovereign default from 0.03% to 0.46%. I calculate that the welfare gains of implementing optimal borrowing policies would have been equivalent to an increase of 0.41% in aggregate consumption. Finally, I prove that optimal private borrowing policies could have been implemented by pairing public debt management with macroprudential taxes on private debt. I estimate an average value of 5% for this tax during the 2008 to 2015 crisis years.

To compute these answers, I combine a dynamic stochastic general equilibrium (DSGE) model of financial crisis caused by collateral debt constraints developed by Mendoza (2002) and Bianchi (2011) with a sovereign debt structure in the tradition of Eaton and Gersovitz (1981) and Arellano (2008).[4] The baseline model consists of a continuum of competitive identical households, international risk-neutral lenders, and a benevolent government. Households consume tradable and nontradable goods and smooth their consumption by borrowing internationally up to a fraction of the market value of their current income. The government has access to lump-sum transfers and strategically defaultable international public debt. Each period, the government chooses to either default on its debt, tax households to pay back part of its debt, or alternatively increase its debt and transfer resources to the households (bailouts). Both private and public liabilities are priced by competitive, risk-neutral international lenders. I allow for exogenous financial shocks to the households' borrowing capacity, the price of private bonds, and the endowments. I then contrast the competitive equilibrium allocations with those chosen by a benevolent social planner. I assume that the planner makes aggregate borrowing decisions on both assets and then transfers the proceeds to the households who make all consumption choices. As a result, the planner and the competitive households are subject to the same market prices, market clearing conditions and credit constraints. Nevertheless, the planner's choice of allocations is different from that of the households because it internalizes the general equi-

[4] See also Kehoe and Levine (1993) for earlier implementations of collateral debt constraints in a general equilibrium context and Aguiar and Gopinath (2006) for early quantitative adaptations of the Eaton and Gersovitz (1981) model.

librium effects of the aggregate choices it makes. Relative to the baseline, the planner chooses a lower level of private debt and achieves a higher level of welfare. I later show that the planner's allocations can be decentralized by extending the baseline framework to allow the government to impose state-dependent taxes on private borrowing.

The first mechanism of the model is Irving Fisher's classic debt-deflation effect. This effect is present in models with a collateral constraint that depends on market-determined prices. Consider the situation of a representative household that enters the period with a large amount of private debt and faces an adverse shock in the form of a productivity or external financial shock. Without government transfers, the household is unable to roll over its debt without violating the collateral constraint. Under these circumstances, the household reacts by reducing both its consumption and its debt. Since all households are assumed to be identical, the reduction in aggregate consumption of tradables induces a decline in the market price of the nontradable goods that causes a decline in the value of collateral. Thus, the credit constraint tightens even more, resulting in an even greater contraction in consumption. The engine of this feedback loop is a general equilibrium price that competitive households take as given. As a result, their individually optimal borrowing decisions are frequently above the socially optimal level. This exposes them to more frequent and severe credit boom and bust cycles relative to a planner who incorporates the general equilibrium effects in its decision making. This financial amplification mechanism is described in Mendoza (2002) and Bianchi (2011) and is referred to as Fisherian deflation.

The novel mechanism of this paper is how this financial amplification interacts with the government's borrowing and default decisions. A benevolent and strategic government will use international public debt to mitigate the most negative consequences of this systemic vulnerability. To fix ideas, I divide the government responses into ex ante episodes, decisions made during periods where the credit constraint is not binding even in the absence of government interventions, and ex post episodes, decisions made during periods when government inaction implies a binding constraint and a contraction in consumption. In all cases, the government evaluates the benefits of providing a positive transfer to households financed with external public debt against the expected costs of a future with either higher taxes or the deadweight losses of a sovereign default.

In an ex ante environment, households' response to a positive transfer financed with debt is subject to the classic consumption-smoothing and Ricardian equivalence effects. Private and public debt behave as substitutes. Households equate the marginal benefit of an additional unit of consumption with the marginal cost of reducing consumption in the future as a result of higher taxes. Consequently, households respond to government transfers by decreasing their private borrowing. This response implies that consumption and total indebtedness remain roughly constant. Depending on the relative price of both assets, the government can find it optimal to substitute some private liabilities with public debt.[5] Since the main consequence of high private debt is an increasing probability of a binding credit constraint in the future, the incentive to intervene in this fashion increases with the level of private debt. As a result, these bailouts are commonly seen in the periods immediately preceding a crisis and when default costs are high.[6] Finally, since the benefits from these interventions are quantitatively small, they are not usually associated with significant increases in interest rate spreads. This mechanism helps the model fit the patterns observed in the data in the years immediately preceding the crisis.

In an ex post scenario, public and private debt behave instead like complements. In these cases, households are facing a binding constraint, and therefore their Euler equation holds with an inequality. In other words, the marginal benefit of current consumption exceeds the marginal cost of lower future consumption. Thus, a positive fiscal transfer in this context always translates into higher individual consumption. Moreover, at the aggregate level, an increase in consumption boosts the relative price of nontradables and with it the value of private collateral. The increasing valuation of collateral brings the opportunity for a higher level of private debt that in turn translates into an additional increase in consumption. This positive financial amplification continues as long as the households' Euler equation holds with an inequality and, as such, facilitates consumption smoothing. However, debt-financed bailouts achieve these gains at the cost of increasing overall indebtedness, since both private and public debt levels rise in tandem. More indebtedness makes defaulting on public debt more appealing. This increases the risk of default in the future and leads to an increase in the interest rate spread paid on public debt in the current period. Quantitatively, the multiplicative benefits of these interventions are large enough to justify the cost of significant increases in spreads. This is the main channel that allows the model to replicate the patterns observed during the peak years of the crisis.

The benefits of restoring the socially efficient level of private debt in this context are twofold. First, by decreasing the level of private borrowing, the planner decreases the severity and frequency of a private financial crisis. Lower private debt implies fewer episodes with a binding credit constraint and contractions in private credit and output. Second, and because of the first benefit, a lower level of private borrowing reduces the need for government bailouts in both ex ante and ex post episodes. Fewer interventions translate into lower public debt and a smaller probability of a sovereign default. The combination of these two factors implies lower interest rate spreads on public debt. I show that macroprudential policies, equated in this paper to taxes on private borrowing, allow the government to decentralize the socially efficient level of private borrowing without distorting optimal public debt management.

The baseline version of the model is calibrated to Spanish data from 1999 to 2011. The calibration targets the mean and the volatility of the private and total debt as well as the interest rate spreads on

[5]The price of government debt depends on expectations of future default costs. Default costs are usually assumed to be proportional to income in the sovereign default literature. Thus, the persistence of income is passed on to default costs.

[6]In practice, since default costs are assumed be increasing in output this can episodes can coincide with output booms.

public debt. I then use the calibrated parameters to solve the socially planned version of the model. I compare the behavior of this counterfactual socially planned economy and the baseline model at their respective ergodic distributions. Taking differences between them, I measure the level of excessive debt, the welfare gains, and the change in the probabilities of experiencing a financial crisis. Finally, I conduct two numerical exercises to evaluate the model's dynamics. Namely, I use the 2008-2015 Spanish data to simulate the crisis in the model. The first exercise is a partial out-of-sample-validation of the modeling approach. I feed into the model the exogenous output and incorporate private default shocks directly from the data. Since financial shocks are unobserved in the data, I use the particle filter approach proposed in Bocola and Dovis (2019) to infer them. I then let the model endogenously generate private and public borrowing and the interest rate spreads. The baseline model replicates the dynamics of private and public debt, bailouts, and spikes on interest rates during the period of interest. Facing the same shocks, the socially planned economy completely avoids an increase in the interest rate paid on public debt through a combination of low private and public debt. Since the interest rate spreads are driven by both private and public debt, in the second exercise I impose as an additional restriction that the path of public debt must coincide exactly with the one observed in the data. As a result, the difference between the interest rate spreads measured in the data and in the counterfactual socially planned economy can be directly attributed to excessive private debt. I use this difference as my conservative estimate of the reduction in spreads at the peak of the crisis in 2012.

Related Literature This paper builds upon the literature on sovereign debt as well as the literature on pecuniary externalities and macroprudential policies. It is most closely related to the literature analyzing the relation between sovereign debt and the domestic private financial sector.

Following the theoretical framework of sovereign defaultable debt introduced in Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008) developed quantitative models of sovereign debt and business cycles. A growing literature has emerged extending their framework. Chatterjee and Eyigungor (2012) and Hatchondo et al. (2016) highlight the importance of long-term debt in generating dynamics of the interest rate spread that are consistent with the data[7]. The model presented here incorporates these findings by assuming a long-term structure for public debt while keeping, for simplicity, the short-term maturity in private debt.[8] The paper is closely related to the branch of the sovereign debt literature that focuses on the link between sovereign debt and the private economy. In contrast to Mendoza and Yue (2009) and Arellano et al. (2017), the analysis presented here assumes that private agents have access to international credit markets even during sovereign default episodes.

[7]These papers, along with Aguiar et al. (2019) and Hatchondo et al. (2016), discuss the issue of "debt dilution", the time-inconsistency problem that emerges when public defaultable debt is long term. Additional papers discussing responses to the trade-offs involved in maturity structures with long-term debt include among many others, Broner et al. (2013), Chatterjee and Eyigungor (2013), and Bianchi et al. (2018).

[8]The presence of multiple maturities links the paper to literature studying the role of the optimal debt maturity structure, such as Arellano and Ramanarayanan (2012) and Sanchez et al. (2018). This paper differentiates itself from this literature by assuming that the government will not be able to fully control the issuances of short-term private debt.

The article shares this feature with Kaas et al. (2020). The main difference with this recent work is that private debt in my model is inefficiently high from a social perspective, and this inefficiency increases the incidence and magnitude of crises. As a result, the frequency of public bailouts, in response to reductions in the borrowing capacity in the private sector, is an endogenous outcome of the model.

The paper is also related to the literature that studies the trade-offs between centralized international public debt and decentralized international private debt. With complete markets, Jeske (2006) finds that a centralized environment, where only the government can issue and default on international debt, allows for more debt and is preferable to a decentralized environment where individual households make the borrowing and default choices. Wright (2006) finds that decentralized private borrowing can lead to underborrowing unless the government can choose to default on behalf of all residents, in which case overborrowing emerges instead. Finally, with incomplete markets, Kim and Zhang (2012) find underborrowing in an environment where decentralized households make the borrowing choices and a centralized government makes the default choice for all agents. My paper assumes incomplete markets and two distinct assets: private and public bonds. Only public debt enjoys sovereign immunity, and the government cannot force private agents to default. In my environment, the decentralization of the private bond leads to overborrowing in both assets relative to a centralized environment where a planner chooses the optimal portfolio.[9] Additionally, I find that both assets are used in the centralized environment because they provide insurance in different states.

Furthermore, the paper contributes to the literature on credit frictions, financial crises, and macroprudential policies. In particular, it belongs to the branch on systemic credit risk (see Lorenzoni (2008), Bianchi (2011), and Dávila and Korinek (2018)) and its management with taxes on private borrowing (see Bianchi and Mendoza (2018), Farhi and Werning (2016), and Jeanne and Korinek (2019)).[10] In related work, Arce et al. (2019) show that optimal international reserve accumulation can achieve the same welfare gains as optimal taxes on borrowing.[11] Instead, this paper shows that by themselves, government bailouts financed with external defaultable debt can partially replace optimal macroprudential policies, but they are not sufficient. The role of bailouts in the model is similar to the one found in Bianchi (2016), Keister (2016), and Chari and Kehoe (2016). In contrast to those papers, I distinctly assume here that the bailouts can be paid for with long-term strategically defaultable debt. This feature allows the model to create a path from financial crises to sovereign debt crises, a relationship observed in the data.[12]

[9] In this context, overborrowing also causes an overvalued real exchange rate in the lead-up to a sovereign default crisis. This is consistent with the Twin D phenomenon, defaults and large devaluations happen in tandem, documented and analyzed in Reinhart (2002) and Na et al. (2018).

[10] Another recent strand of related literature studies the implications of this pecuniary externality for exchange rate policy, see, for example Fornaro (2015), Ottonello (2015), and Benigno et al. (2016).

[11] Bianchi (2011) also shows the optimal allocations can be implemented with capital reserve requirements.

[12] The literature on bailouts also deals extensively with the issue of moral hazard that the expectation of government bailouts induces. This concern is not addressed in this paper since households take as given that government policies are functions of aggregate states and not their individual actions. Additional research on the issue can be found in Nosal and Ordoñez (2016), Stavrakeva (2020), and Pasten (2020).

By analyzing how private credit affects the sovereign spread, the paper also contributes to the growing literature on the feedback loop between sovereigns and the domestic financial sector referred to as ”doom loops” or ”lethal embrace”. Theoretical models of this issue are presented in Korinek (2012), Brunnermeier et al. (2016), and Farhi and Tirole (2018).[13] Acharya et al. (2014) present empirical evidence documenting the problem and theoretical model that is qualitatively consistent. The model presented here distinguishes itself by providing theoretical model that is also quantitatively consistent with the Spanish data. With this approach, the paper is more closely related to other quantitative models of the relationship between sovereigns and private borrowers, such as Perez (2015), Bocola (2016), and Sosa-Padilla (2018). The analysis in these papers focuses on one leg of the loop, the role of sovereign debt in the balance sheet of domestic banks and how the increase in sovereign spreads exacerbates domestic credit vulnerabilities. My paper complements their work by instead studying the other leg loop, from financial financial vulnerabilities to sovereign debt crises. In this paper, preexisting private credit vulnerabilities create incentives for government interventions that increase default risk and spreads.

Finally, methodologically the paper applies dynamic discrete choice methods to solve a sovereign debt model drawing from the contributions of Dvorkin et al. (Forthcoming)[14]. The method is used here to smooth the government policy functions and reduce computational errors. Additionally, to construct a quantitative counterfactual of the Spanish debt crisis, the paper uses the nonlinear particle filter method proposed by Kitagawa (1996). This technique uses likelihood functions to construct a numerical approximation of an unobserved stochastic shock and was first applied to quantitative business cycle models in Bocola (2016) and Bocola and Dovis (2019).

Layout. The paper is organized as follows. Section 2 outlines the motivating empirical facts in the Spanish data. Section 3 presents the model and the main theoretical result. Section 4 details the calibration and discusses the main mechanisms through which private and public debt interact in the model. Section 5 provides the quantitative results of the paper. Its first part compares the positive and normative versions of the model at their respective ergodic distributions. Its second part details the two dynamic exercises that simulated the 2008-2015 Spanish debt crisis. It provides the model predictions and counterfactual dynamics for private and official borrowing, and the evolution of interest spreads. Finally, Section 6 concludes.

2 Motivation: The path of debt and spreads in Spain 1999-2015

This section documents the evolution of international private and public debt in Spain from the creation of the Eurozone in 1999 to the end of the Spanish sovereign debt crisis in 2015. The pattern

[13]Other relevant theoretical papers on this issue include Uhlig (2014) and Cooper and Nikolov (2018).

[14]Other models using this technique include Mihalache (2020). A review of the method and an alternative can be found in Gordon (2019).

consists of a period of vast accumulation of private debt, with low levels of public debt and spreads, followed by financial and sovereign debt crises. The evolution of these assets, and that of their underlying default risks, serves as an illustration of the intertwined relationship between private vulnerabilities and sovereign debt crises. I document this pattern for Spain; however, as noted by Reinhart and Rogoff (2011), Lane (2013), and Gennaioli et al. (2018), similar patterns have been seen in other countries and periods.

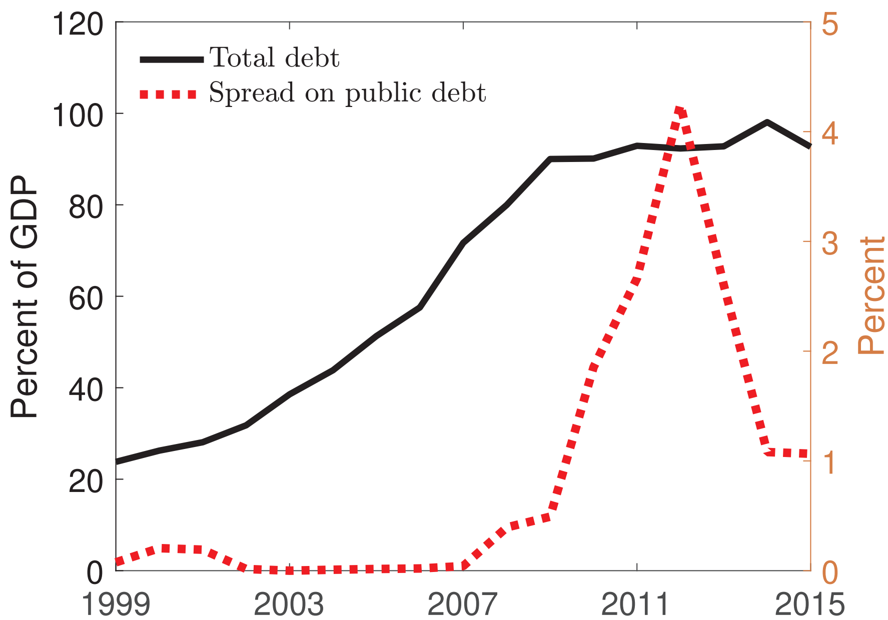

Figure 1: Total international debt and sovereign spread

Note: Total debt corresponds to the inverse of the international investment positions. Spreads correspond to the average difference between the interest rate paid on a Spanish six year treasury bill and its German equivalent. The data source for debt is the Bank of Spain, and the interest rate data are from Bloomberg. More details can be found in Appendix C.

Figure 1 plots the evolution of the Spanish debt crisis and exposes the difficulty of studying external debt without distinguishing between private and public liabilities. The left axis plots the evolution of the international investment position as a percentage of GDP on an inverted scale; that is, positive numbers represent net liabilities.[15] All types of assets are accounted for in this aggregate. Nevertheless, throughout the paper I refer to this measure of net international liabilities as debt. The right axis plots the sovereign spread (dotted line), calculated as the difference between a six-year Treasury bond issued by Spain and its German counterpart.[16] The figure shows a first period of accumulation of external debt between 1999 and 2008, followed by a period where total debt remained constant at around 92% of GDP. These dynamics are juxtaposed with the evolution of the sovereign spread. Interest rate spreads remain close to zero up to 2009 and then experience a spike in 2012. Looking at

[15] Annualized data are from the Bank of Spain; more details can be found in Appendix C.

[16] This maturity is chosen because it corresponds to the average maturity of public debt in Spain during this period. For more details, see Section 4 and Appendix C.

this figure through the prism of the standard sovereign debt model makes it hard to reconcile a period with rapidly increasing debt but low spreads (1999 to 2008) with a period of significant movement in the spread with constant total debt (2009 to 2015). Indeed, Banco de España (2017) argues that the period of near-zero spreads is evidence that financial markets underappreciated risk.

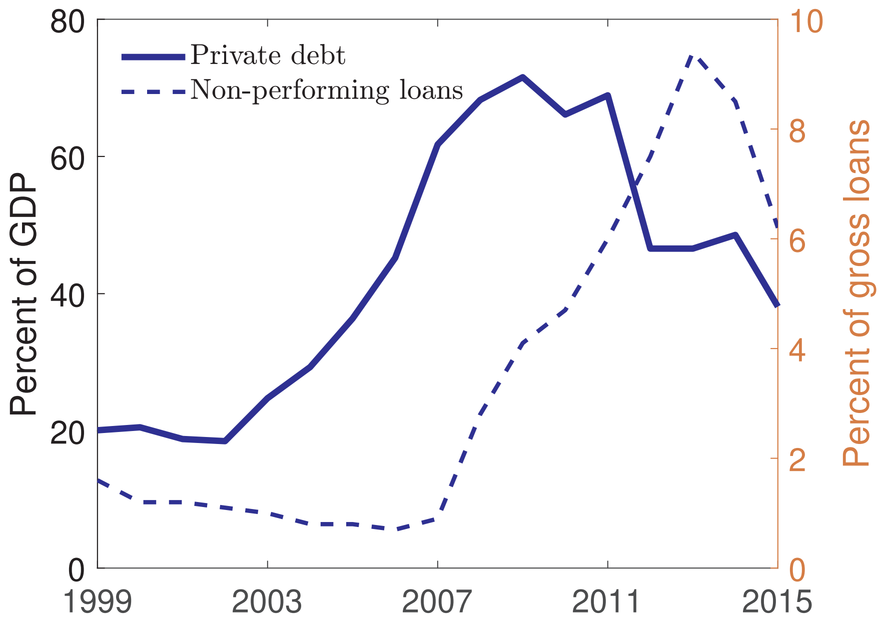

Figure 2: Private debt and nonperforming loans
Note: Private debt corresponds to the inverse of the international investment positions of the financial and nonfinancial private sector. Nonperforming loans are computed as a share of total gross loans. The data source for debt is the Bank of Spain, and the loans data are from Bloomberg. More details can be found in Appendix C.

Next, I summarize in Figure 2 the evolution of the private international liabilities during this time period. The right axis corresponds to the debt position of the private sector as a percentage of GDP (solid line) and the right axis corresponds to nonperforming loans as a percentage of gross loans (dashed line).[17] As before, the evolution of private debt displays two distinct periods. Net liabilities in the private sector grew from 20% of GDP in 1999 to 70% of GDP in 2009. Contemporary observers of this trend, such as International Monetary Fund (2007), classified the growth in private credit as the main risk to Spanish growth but predicted that the imbalances would gradually disappear.[18] After declining slightly for two years, private debt dropped by 22% of GDP in 2012. As noted by International Monetary Fund (2012), International Monetary Fund (2014), and Martin et al. (2019), among others, the buildup of external private debt was primarily driven by a banking sector that was financing a construction boom. When housing prices fell and mortgages started going unpaid, private debt

[17] To compute the position of the private sector, I subtract from total debt the assets held by the public administration and the Bank of Spain. See details in Appendix C.

[18] The empirical literature finds that strong link between international private credit growth and financial crises. See for instance, Schularick and Taylor (2012) and Davis et al. (2016).

became increasingly more difficult to roll over abroad. For this reason, I use the percentage of nonperforming loans as a proxy measure of aggregate default risk in the private sector. Figure 2 shows that the rapid increase in private debt stopped roughly at the same time as the share of nonperforming loans started increasing. Moreover, the abrupt drop in 2012 coincided with a high mark of the share of private default. On average, 7.5% of gross loans were nonperforming between 2011 and 2015.

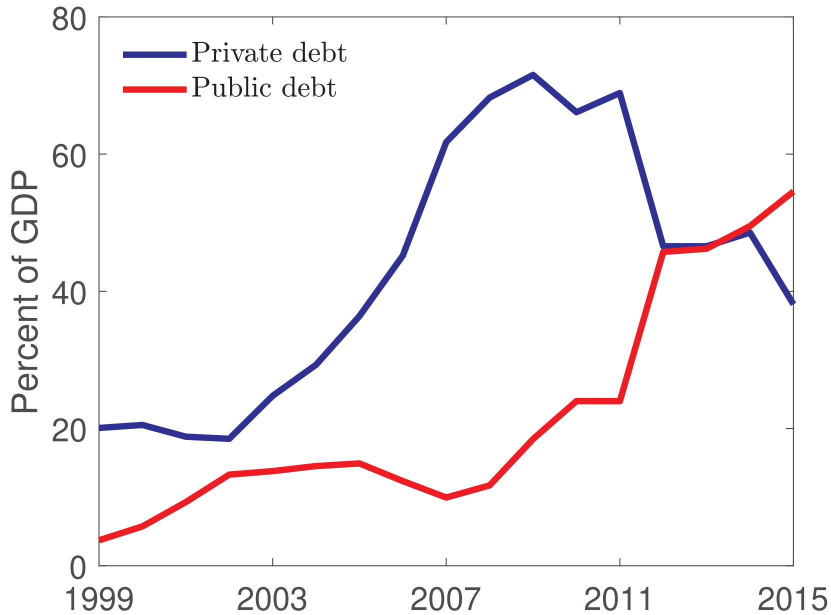

Figure 3: Private and public debt

Note: Private debt corresponds to the inverse of the international investment positions of the financial, and non financial private sector. Public debt corresponds to the inverse of the international investment position of the Bank of Spain and other public administrations. The data source is the Bank of Spain. More details can be found in Appendix C.

Finally Figure 3 complements the analysis by showing the joint evolution of public and private debt. The figure plots the evolution of private debt alongside the evolution of public liabilities. Combined, these two positions add up to the total debt presented in Figure 1. The symmetry between these two aggregates highlights the importance of the decomposition presented in this section. From 1999 to 2008, public external debt in Spain was below 20% of GDP. In contrast, from 2008 to 2015, public external debt increased from 11% to 55% of GDP. More importantly, the largest yearly increase was also in 2012, when public liabilities increased by 22% of GDP, exactly mirroring the drop in private debt. As noted in Banco de España (2017), this symmetry is not a coincidence. Between 2008 and 2012, the Spanish government funneled financial assistance to its lending institution primarily in the forms of bailouts and transfers of toxic assets. Total direct aid to the Spanish banking sector amounted to 70 € billion or around 7% of GDP, with most of these funds being transferred by the newly created Fund for the Orderly Restructuring of the Banking Sector (FROB).[19]

[19] A full overview of the restructuring of the Spanish financial sector is beyond the scope of this paper. More details can be found in International Monetary Fund (2010) and Banco de España (2017).

In the pre-crisis years, 1999-2007, large buildups of private debt coexist with low public debt and public spreads close to zero. This period is followed by a private financial crisis, corresponding in the data to the years 2008 to 2011. The financial crisis is characterized by an increase in nonperforming loans in the private sector and a moderate private deleveraging. Throughout this period, public debt and spreads increase but remain relatively low. The final period, from 2012 to 2015, corresponds to the sovereign debt crisis. These years are characterized by large public bailouts that reduce net liabilities in the private sector but are financed with issuances of public debt. The shift in debt ownership coincides with significant increases in the spread paid on public debt. The next section proposes a theory that sheds light on this interplay between private and public external debt. The goal is to construct a model where both types of debt and their prices are endogenous, and that generates dynamics consistent with the facts presented in this section.

3 A model of financial and sovereign debt crises

This section presents a dynamic small open-economy model with one-period international private bonds subject to an occasionally binding borrowing constraint as in Bianchi (2011), and long-term, strategically defaultable, international public bonds, as in Hatchondo and Martinez (2009). The first subsection presents the economy's environment and technologies. The second subsection defines and characterizes the baseline unregulated, competitive equilibrium where the government only has access to public debt and lump-sum transfers. The third shows the optimal policy problem of a social planner (SP) who makes all borrowing decisions in both assets. The fourth subsection demonstrates that the SP's allocations are equivalent to those of a competitive equilibrium where the government gains access to state-contingent taxes on private debt. The last subsection explains the main mechanism of the model.

3.1 Environment

Time is discrete and indexed by $t \in \{0, 1, ..., \infty\}$. The economy is composed of a continuum of identical households of unit measure, a benevolent domestic government, and a continuum of risk-neutral competitive foreign creditors who lend to both domestic agents via two different assets. The focus is on real values as opposed to nominal ones because most Spanish debt was denominated in euros[20].

3.1.1 Households

Preferences The representative household has an infinite life horizon and preferences given by

[20] The interaction of sovereign default and the inability to inflate away the debt in the context of the European Debt Crisis is studied in Aguiar et al. (2014) and Aguiar et al. (2015). For the specific case of Spain, Bianchi and Mondragon (2018) explore this issue in an environment with nominal rigidities.

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + D_t], \tag{1}$$

where $\mathbb{E}_0$ is the expectation operator conditional on date 0 information; $0 < \beta < 1$ is a discount factor; and $u(\cdot)$ is a standard increasing, concave, and twice continuously differentiable function satisfying the Inada condition. The term D_t is an additive preference shifter that depends entirely on government decisions, and exogenous shocks and the households take it as given. The consumption basket c is an Armington-type constant elasticity of substitution (CES) aggregator with elasticity of substitution $1/(\eta + 1)$ between tradable goods c^T and nontradable goods c^N, given by

$$c = \left[\omega \left(c^T\right)^{-\eta} + (1-\omega)\left(c^N\right)^{-\eta}\right]^{-\frac{1}{\eta}}, \eta > -1, \omega \in (0,1).$$

Endowments Each period the economy receives a stochastic endowment of tradable goods $y^T \in \mathbb{R}^+$ and nontradable goods $y^N \in \mathbb{R}^+$. Both endowments are drawn from first-order Markov processes independent of each other and of all other stochastic shocks in the model. The numeraire is the tradable good.

Private Debt Households can borrow using a one-period non-state-contingent debt denominated in units of tradables. Following the standard convention, lowercase b denotes the individual level of private debt, while uppercase B denotes the aggregate level. Each period a stochastic fraction π_t of these bonds is defaulted on. Including these private default shocks allows the model to capture the dynamics of nonperforming loans in Spain and imply a more realistic cost of borrowing for private debt. Like the endowment shocks, the fraction of defaulted private bonds is drawn from a first-order Markov process independently from all the other stochastic shocks in the model. Considering this, private debt is issued in international competitive credit markets at price q_t. In equilibrium, q_t depends on the expected future private default shocks. In addition, private bonds are subject to a collateral credit constraint. The market value private debt issuances $q_t b_{t+1}$ is capped at a fraction $\kappa_t \geq 0$ of the market value of current income:

$$q_t b_{t+1} \leq \kappa_t \left(y_t^T + p_t^N y^N\right), \tag{2}$$

where p_t^N is the equilibrium price of nontradable goods in units of tradables. This credit constraint captures in a parsimonious way the empirical fact that income is critical in determining credit market access.[21] Theoretically, the constraint can be derived as an implication of incentive-compatibility constraints on borrowers if limited enforcement prevents lenders from collecting more than a frac-

[21]See Jappelli (1990).

tion κ_t of the value of the endowment owned by a defaulting household.[22] Nontradable goods enter the collateral constraint because even though foreign creditors do not value them, I assume they can be seized in the event of default and sold in exchange for tradable goods in the domestic market.[23] Empirically, collateral constraints are commonly used in mortgage lending. Consequently, this assumption is particularly suitable in the Spanish context where mortgage loans played an important role in the buildup of private credit. Note that, while private debt is explicitly modeled here as issued internationally by the households, the same constraint arises under a broader set of assumptions. In particular, I could assume instead that credit is provided to households by a competitive domestic financial system with unrestricted access to global capital markets but subject to the same enforcement friction. As noted in Section 2, this interpretation is more in line with the events that unfolded in Spain. Commercial and savings banks borrowed internationally and then channeled these funds to households and construction firms. The assumption of short-term maturity is consistent with the empirical literature documenting a reduction in the maturity of private bonds issued in advanced economies during this period.[24]

The fraction of market income required as collateral κ_t is stochastic and drawn from a first-order Markov process. Throughout the paper, I refer to this shock as the financial shock. Stochastic changes in collateral requirements can be viewed as shocks to the creditors' risk assessment of the borrowers. Financial shocks of this form have been shown to be capable of accounting for the dynamics of private financial crises in advanced economies (see Jermann and Quadrini (2012), and Boz and Mendoza (2014)) as well as balance of payment crises in emerging economies (see Mendoza (2002) and Coulibaly (2018)). From a modeling perspective, these shocks generate fluctuations in private borrowing that are not caused by fluctuations in domestic fundamentals. This is consistent with recent empirical work by Forbes and Warnock (2020). They document that shocks in international volatility, monetary policy, or sudden-stop crises in similar and/or neighboring countries can cause fluctuations in the lenders' perceptions about the private sector's solvency. In the context of interest, these shocks allow the model to account for a change in investors' behavior toward Eurozone banks in the wake of the Greek sovereign debt crisis.

To conclude, note that neither the existence of the financial amplification mechanism nor the government's best responses presented later rely on κ_t or π_t being stochastic.[25] Nevertheless, these shocks will generate fluctuations in private borrowing independently from income fluctuations and as such will have a different impact on government policies.

[22] In this context, the punishment is only triggered by private default above the exogenous fraction drawn in each period.

[23] The current, rather than the future, price appears in the constraint because the opportunity to default occurs at the end of the current period, before the realization of future shocks. See Bianchi and Mendoza (2018), for a derivation of a similar constraint.

[24] See, for instance, Gorton et al. (2020) and Chen et al. (2019).

[25] Models with a constant κ and no private default, such as Mendoza (2010), also generate private crisis dynamics with realistic business cycle features.

Households' budget constraint Each period, individual households face a budget constraint of the form

$$(1 - \pi_t)b_t + c_t^T + p_t^N c_t^N = q_t b_{t+1}, +y_t^T + p_t^N y^N + T_t. \tag{3}$$

where T_t is a lump-sum transfer from the government. A positive transfer indicates a bailout, while a negative one denotes a lump-sum tax. This transfer is the primary link between the households and the government and will be present in all versions of the model. Access to this instrument allows the government to directly modify the household's cash-in-hand without introducing additional distortions. As a result, the interactions that will arise between private and public debt in this paper are not a consequence of a restrictive set of tax instruments. In contrast, in models with distortionary taxes, the main mechanism of this paper will interact with the distortions introduced by the functional form of the tax instrument. The last subsection will consider the implications of giving the government an additional tax instrument, a linear tax on private borrowing, τ_t, used for macroprudential purposes.

3.1.2 Government

Public debt The government borrows by issuing without commitment a long-term bond ($L \geq 0$) on international capital markets *à la* Eaton and Gersovitz (1981). Each period the sovereign chooses to either default ($d \in \{0, 1\}$) or to keep its credit market access by paying its obligations and reissuing new ones. As in Arellano and Ramanarayanan (2012) and Hatchondo and Martinez (2009), I assume that a bond issued in period t promises in case of repayment a deterministic infinite stream of coupons that decreases at an exogenous constant rate δ. As such, one unit issued in the current period promises to pay a fraction $(1 - \delta)$ of all remaining debt each following period. An advantage of this payment structure is that it condenses all future payment obligations into a one-dimensional state variable proportional to the quantity of long-term coupon obligations that mature in the current period. Hence, the debt dynamics can be summarized by

$$L_{t+1} = (1 - \delta)L_t + i_t, \tag{4}$$

where L_t is the number of public bonds due at the beginning of period t, and i_t is the bond issuances at t. As in common in the literature, I assume that sovereign debt only takes values in a finite and bounded support with $\mathcal{J}$ points.[26] The grid of potential long-term debt positions can be summarized by a vector Λ, where L_j is the *jth* element, consequently,

$$\Lambda = \left[L_1, L_2, ..L_{\mathcal{J}}\right]^T.$$

[26]The assumption of a discrete and bounded support is usual in the sovereign default literature with long-term debt; see Chatterjee and Eyigungor (2012).

Default Default brings immediate financial autarky and an additive utility cost that is an increasing function of tradable output $\phi(y_t^T)$. [27] For simplicity, I assume that the government returns to international credit markets with zero debt after one period of exclusion from markets.[28] Note that sovereign default does not imply default on private debt nor an exclusion of private agents from financial markets. This is in contrast to other papers with both public and private international debt, such as Mendoza and Yue (2009). I make this assumption for both empirical and conceptual reasons. Empirically, Kalemli-Ozcan et al. (2018), Gennaioli et al. (2018), and Bottero et al. (2020) find that although private borrowing declines during a sovereign default crisis, it does not disappear. Conceptually, this paper focuses on endogenizing the interaction between the two types of debt and assuming that joint default imposes a direct effect between them.

Government's preferences The sovereign is benevolent and therefore has the same utility and discount factors as the households. Furthermore, for computational tractability, I follow Dvorkin et al. (Forthcoming) and assume that each period the government draws a random vector ϵ of size $\mathcal{J}+1$ of additive taste shocks. One element of the vector is associated with the choice of default, while the remaining $\mathcal{J}$ elements are associated with each debt choice on Λ in case of repayment. The elements of the vector are labeled

$$\epsilon(L_j) = \epsilon_j,$$
$$\epsilon^{Def} = \epsilon_{\mathcal{J}+1}.$$

The taste shock ϵ is independent and identically distributed (i.i.d.) over time and within Λ. Furthermore, I assume that its distribution is a multivariate generalized extreme value with mean m and variance $v > 0$.[29] Preference shocks affecting the default decisions are now common in the literature; see, for instance, Arellano et al. (2017), Aguiar et al. (2019), and Aguiar et al. (2020). They are considered an alternative to the i.i.d. income shocks also encountered in the literature (e.g., Chatterjee and Eyigungor (2012)). In this model, the shocks allow the government to break ties between similar portfolio positions. An interpretation of these shocks is that they capture additional costs or benefits of default, such as the perceptions of policy makers of the costs of default. At the same time, as noted by Dvorkin et al. (Forthcoming), provided that the variance of the shocks is small enough, they will have small quantitative consequences in aggregate moments. Combining all this, the government's flow utility at time t is

[27] Utility losses from default in sovereign debt models are also used in Aguiar and Amador (2013), Bianchi and Sosa-Padilla (2020), and Roch and Uhlig (2018), among others. A common alternative is output costs of default. If the utility function is log over the composite consumption, and output losses from default are proportional to the composite consumption in default, the losses from default would be identical across the two specifications.

[28] Assuming an exogenous probability of reentry into financial markets, as in Arellano (2008), would not change the results but would require to keeping track of an additional state.

[29] For additional details regarding the distribution of taste shocks, see Appendix A.

$$u(C_t) + d_t\big(\epsilon_t^{Def} - \phi(y_t^T)\big) + (1-d_t)\epsilon_t(L_{t+1}),$$

where d_t is the government default decision, C_t is private consumption, $\phi(y_t)$ is the utility cost of default, and ϵ_t is the additive taste shock. This equation provides an explicit formulation of the additive preference term in the household preferences (1), namely,

$$D_t = d_t\big(\epsilon_t^{Def} - \phi(y_t^T)\big) + (1-d_t)\epsilon_t(L_{t+1}).$$

Government's budget constraint Each period the government's budget constraint is given by its default decision d_t, the public debt dynamics (4), and the lump-sum transfers T_t.[30] The budget constraint is

$$T_t = (1-d_t)\Big[Q_t\big[L_{t+1} - (1-\delta)L_t\big] - \delta L_t\Big], \tag{5}$$

where L_t is the long-term public debt at the beginning of period t, and L_{t+1} is the long-term debt at the end. Finally, Q_t is the price at which lenders purchase these bonds, which in equilibrium depends on the government's and household's portfolio decisions and the exogenous shocks.

3.1.3 International lenders

Private and sovereign bonds are traded with a continuum of risk-neutral, competitive foreign lenders. Lenders have access to a one-period risk-free security paying a net interest rate r. The equilibrium price of private bonds is given by the no-arbitrage condition

$$q_t = \frac{\mathbb{E}_t\left[1-\pi_{t+1}\right]}{1+r}.$$

In equilibrium, investors must be indifferent between purchasing a risk-free security and buying a private bond at price q_t. Since private debt is only held for one period, lenders use the exogenous probability of default one period ahead to price it. Similarly, bond prices for sovereign debt in case of repayment are

$$Q_t = \frac{\mathbb{E}_t}{1+r}\Big[(1-d_{t+1})\big(\delta + (1-\delta)Q_{t+1}\big)\Big].$$

As before, the no-arbitrage condition implies that investors will purchase government bonds at a price Q_t that compensates them for the risk of default they bear. In case of default, no public debt is recovered. In case of repayment, the payoff is given by the coupon δ plus the market value Q_{t+1} of the nonmaturing fraction of the bonds next period.

[30] The last subsection will modify this constraint by granting the government access to taxes on private debt.

3.1.4 Resource constraints

Since both types of debt are denominated in tradables, the market clearing conditions are

$$c_t^N = y_t^N, \tag{6}$$

$$c_t^T + (1 - \pi_t) b_t = y_t^T + q_t b_{t+1} + T_t. \tag{7}$$

3.2 Baseline unregulated competitive equilibrium

This subsection defines and characterizes the baseline problem in recursive form. I first discuss the equilibrium concept and the timing of the events and introduce the notation used throughout the paper. I then present, in order, the problems of the government, the households, and the lenders. I conclude with the formal definition of a competitive equilibrium for this baseline version of the model.

Equilibrium concept This paper will focus on a Markov perfect equilibrium. Consequently, the current period decisions of all agents will be functions of payoff-relevant state variables and will take all future policies rules as given. The focus on a Markov perfect equilibrium is important. An environment with strategically defaultable long-term bonds with a government that cannot commit to future debt issuances induces a time-inconsistency problem known as debt dilution. The solutions to the recursive, time-consistent problem do not coincide with the solutions to the sequential problem with commitment. Throughout the paper, the focus is on the time-consistent policies.[31] Consequently, government default, borrowing and transfer strategies each period will only depend on current period payoff-relevant states.

One could interpret this environment as a game where the government makes current period decisions while taking as given the best response functions of the other players, households, and foreign lenders, and also the strategies of future governments that decide policies in the future. Thus, the government considers the general equilibrium effects of its policies on the aggregate choices of the private sector, consumption, and private borrowing, and all prices, nontradables, and bonds, but cannot choose those functions.

Recursive notation and timing In all cases, I denote with a prime symbol the end-of-period levels of private and public debt. The timing of events within the period is as follows:

- The economy enters the period with private debt B and public debt L.
- All shocks are realized. The exogenous state is $s = \{y^T, y^N, \kappa, \pi, \epsilon\}$.

[31] For a discussion of policies that remedy debt dilution, see Hatchondo et al. (2016) and Aguiar et al. (2019).

- The state space is now $S = \{s, L, B\}$.
- The government acts first. Facing S, the government makes default d and public debt L' choices.
- The aggregate state of the economy incorporating the government's policies is $S_G = \{S, d, L'\}$.
- Households act second. Facing S_G, households choose consumption and private debt, which determine the aggregate consumption C^T and C^N and the aggregate private debt B'.
- The lenders act last. They choose bond schedules Q and q using only the payoff-relevant states.

Policy decisions and best responses The government's policy decisions are $\boldsymbol{d}(S), \mathcal{L}'(S)$. The private sector's aggregate best responses are $\mathcal{C}^T(S_G)$, $\mathcal{C}^N(S_G)$, and $\mathcal{B}'(S_G)$. The foreign lenders' best responses are the schedules for public bond $Q(s, L', \mathcal{B}'(S_G))$ and for private bond $q(s)$.

Government The government is a strategic player. Given the best responses of the private sector and foreign lenders, the government chooses $\boldsymbol{d}(S)$ and $\mathcal{L}'(S)$ that maximizes the household's welfare subject to the period budget constraint (5) and the resource constraints, (6) and (7). In detail, the government's problem is

$$W(S) = \max_{d\in\{0,1\}} [1-d]W^R(S) + dW^D(S), \tag{8}$$

where $d = 1$ if the government defaults and $d = 0$ otherwise. If the government repays, $S_G = (S, 0, L')$, and the value of repayment is

$$W^R(S) = \max_{L'\in\Lambda} \; u\big(\mathcal{C}^T(S_G),\mathcal{C}^N(S_G)\big) + \epsilon(L') + \beta\mathbb{E}_s[W(s', L', \mathcal{B}'(S_G))] \tag{9}$$

$$\begin{aligned} &\text{subject to} \\ T(S_G) &= Q(s, L', \mathcal{B}'(S_G))[L' - (1-\delta)L] - \delta L \\ \mathcal{C}^T(S_G) + (1-\pi)B &= y^T + q(s)\mathcal{B}'(S_G) + T(S_G), \\ \mathcal{C}^N(S_G) &= y^N. \end{aligned}$$

Note that in repayment states, the government's public debt decision will affect the value of the transfer directly through issuances and indirectly through the bond schedule. The choice of public debt will then affect the households' decisions on consumption of tradables and private debt via the transfer. The government internalizes that its borrowing decision affects the choices of the households and the price that the lenders will charge for public debt.

In the case of default, $S_G = (S, 1, 0)$, and the government's value is

$$W^D(S) = u\Big(C^T(S_G), C^N(S_G)\Big) + \epsilon^{Def} - \phi(y^T) + \beta \mathbb{E}_s\Big[W\big(s', 0, B'(S_G)\big)\Big] \tag{10}$$

subject to

$$T = 0,$$

$$C^T(S_G) + (1-\pi)B = y^T + q(s)\mathcal{B}'(S_G),$$

$$C^N(S_G) = y^N,$$

In default, the government loses access to public borrowing. Thus, the transfer is zero. Nevertheless, households still maintain access to financial markets and are still liable for their obligations. Consequently, a sovereign default can still leave the economy highly leveraged, albeit in private bonds.[32]

The solution to the government's problem yields decision rules for default $\boldsymbol{d}(S)$ and public debt $\mathcal{L}'(S)$, which in turn determine the transfers $T(S_G)$ and the preference shift $D(S_G)$ as follows:

$$T(S_G) = (1 - \boldsymbol{d}(S)) \times \Big(Q(s, \mathcal{L}'(S), \mathcal{B}'(S_G))[\mathcal{L}'(S) - (1-\delta)L] - \delta L\Big) \tag{11}$$

$$D(S_G) = \big(1 - \boldsymbol{d}(S)\big)\epsilon\big(\mathcal{L}'(S)\big) + \boldsymbol{d}(S)\big(\epsilon^{def} - \phi(y^t)\big) \tag{12}$$

Households The households make decisions based on their current level of individual debt b and the aggregate state of the economy when they act, S_G. The aggregate state comprises the exogenous shocks s, the initial level of government debt L, the current level of aggregate private debt B, and the decisions made by the government in the current period regarding default d and public debt L'. Households are competitive, and as such they take all prices and aggregate laws of motion as given: the price of nontradables $p^N(S_G)$, the equilibrium price of private bonds $q(s)$, the government's current and all future borrowing decisions $\mathcal{L}'$ and default decisions $\boldsymbol{d}$,[33] transfers T, and the preference shock D. Under rational expectations, households predict future states using the perceived law of motion of

[32]See Mendoza and Yue (2009) for a case where public default also triggers private default.

[33]For concision, I equate in the discussion the solutions to the current government policy functions with the strategies of future governments. This equality holds in a Markov perfect equilibrium. Alternatively, one could impose this equality as an equilibrium condition, as in Bianchi and Mendoza (2018).

aggregate private debt $\mathcal{B}'$. The households' optimization problem in recursive form is

$$V(S_G, b) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + D + \beta \mathbb{E}_s[V(S'_G, b')] \tag{13}$$

$$\text{subject to}$$

$$\begin{aligned} c^T + p^N(S_G)c^N + (1-\pi)b &= y^T + p^N(S_G)y^N + q(s)b' + T, \\ q(s)b' &\leq \kappa[p^N(S_G)y^N + y^T], \\ T &= T(S_G), \\ D &= D(S_G), \\ B' &= \mathcal{B}'(S_G), \\ L' &= \mathcal{L}'(S), \\ \text{And } S'_G &= (s', L', B', \boldsymbol{d}(s', L', B'), \mathcal{L}'(s', L', B')). \end{aligned}$$

In equilibrium, $p^N(S_G)$ is the price of nontradables, and $q(s)$ is the price of private bonds. The solution to the household problem yields decision rules for individual bond holdings $\hat{b}'(S_G, b)$, tradable consumption $\hat{c}^T(S_G, b)$, and non-tradable consumption $\hat{c}^N(S_G, b)$. The household optimization problem induces a mapping from the perceived law of motion for aggregate bond holdings, $\mathcal{B}'(S_G)$, to an actual law of motion, given the representative agent's choice $\hat{b}'(S_G, B)$. In a rational expectations equilibrium, these two functions must coincide. The same is true for the laws of motion of aggregate consumption in the economy $\{C^i(s, L, B)\}_{i=T,N}$.

The solutions to the households' problem solve the optimality conditions that include the budget constraint (3), the credit constraint (2), and the first-order conditions. In particular, the households' intratemporal optimality condition pins down the equilibrium price of nontradables,

$$p^N(S_G) = \frac{1-\omega}{\omega} \left(\frac{C^T(S_G}{y^N} \right)^{\eta+1}. \tag{14}$$

Condition (14) is a static optimality condition equating the marginal rate of substitution between tradable and nontradable goods to their relative price. The equation implies that the price of nontradables is an increasing function of c^T. A pecuniary externality arises in this problem because this equilibrium price affects the value of collateral (2) and therefore the level of borrowing in some states. Consequently, a reduction in c^T causes in equilibrium a reduction in the collateral value (2). In states where the credit constraint binds, this reduction triggers the financial amplification mechanism, whereby a drop in consumption induces a contraction in private borrowing, which in turn drives consumption further down. Because of standard consumption-smoothing effects, consumption increases with the cash-in hand of the households. Since the government can increase the cash-in-hand of the households via the fiscal transfer, mitigating the amplification mechanism is an important incentive for government bailouts.

Lenders The competitive risk-neutral foreign lenders use the decision rules of current and future governments and households to price debt bonds. The solution to the problem of competitive risk-neutral foreign lenders yields the bond price schedule for private debt,

$$q(s) = \frac{\mathbb{E}_s[1 - \pi']}{1 + r}, \tag{15}$$

and the bond price schedule for public debt

$$Q(s, L', B') = \frac{1}{1 + r} \times \mathbb{E}_s\left[\left[1 - d'\right] \times \left[\delta + (1 - \delta)Q\left(s', L'', B''\right)\right]\right], \tag{16}$$

$$\begin{aligned} &\text{Where:} \\ B'' &= \mathcal{B}'(s', L', B'), \\ L'' &= \mathcal{L}'(s', L', B'), \\ d' &= \boldsymbol{d}(s', L', B'). \end{aligned}$$

The lenders price the debt contracts based on their expectations of future defaults and new issuances of public debt. As a result, when pricing private debt, the only payoff-relevant state is the exogenous shock s. In contrast, when pricing public debt, the payoff-relevant states for the lenders also include the end-of-period levels of private B' and public debt L'. Note that both the levels and composition of debt are important because they affect the future governments' default and public debt issuances decisions.

Definition of equilibrium The competitive Markov equilibrium combines the problems of the government, households, and lenders, as well as the resource constraints of the economy. Moreover, it also has rational expectations conditions guaranteeing that in equilibrium the households' borrowing and consumption decisions are consistent with the perceived law of motion that all agents are using in their decisions.

Definition 1. *A Markov unregulated competitive equilibrium is a set of value functions* $\{V, W, W^R, W^D\}$*, policy functions for the private sector* $\{\hat{b}, \hat{c}^T, \hat{c}^N\}$*, policy functions for the public sector* $\{\boldsymbol{d}, \mathcal{L}'\}$*, a pricing function for nontradable goods* p^N*, pricing functions for public debt* Q *and private debt* q*, and perceived laws of motion* $\{\mathcal{B}', C^T, C^N, Q\}$ *such that*

1. *Given prices* $\{p^N, q\}$*, government policies* $\{\boldsymbol{d}, \mathcal{L}'\}$*, and perceived law of motion* $\mathcal{B}'$*, the private policy functions* $\{\hat{b}', \hat{c}^T, \hat{c}^N\}$ *and value function* V *solve the household's problem (13).*

2. *Given bond prices* $\{Q, q\}$ *and aggregate laws of motion* $\{\mathcal{B}', C^T, C^N\}$*, the public policy functions* $\{\boldsymbol{d}, \mathcal{L}'\}$ *and value functions* W*,* W^R*, and* W^D *solve the Bellman equations (8)–(9).*

3. *Households' rational expectations: perceived laws of motion are consistent with the actual laws of motion* $\{\mathcal{B}'(S_G) = \hat{b}'(S_G, B), C^T(S_G) = \hat{c}^T(S_G, B), C^N(S_G) = \hat{c}^N(S_G, B)\}$.

4. *The private bond price function* $q(s)$ *satisfies (15).*

5. *Given public* $\{\boldsymbol{d}, \mathcal{L}'\}$ *and private* $\{\mathcal{B}'\}$ *policies the public bond price* $Q(s, L', B')$ *satisfies (16).*

6. *Goods market clear:*

$$C^N(S_G) = y^N,$$
$$C^T(S_G) + (1-\pi)B = y^T + q(s)\mathcal{B}'(S_G) + \Big\{1 - \boldsymbol{d}(S)\Big\}\Big\{Q(s, \mathcal{L}'(S), \mathcal{B}'(S_G))\Big[\mathcal{L}'(S) - (1-\delta)L\Big] - \delta L\Big\}.$$

3.3 Recursive social planner's problem

This subsection formulates the problem of a social planner (SP) in the same environment. The formulation is similar to the "primal approach" to optimal policy analysis. The planner chooses aggregate allocations subject to resource, implementability, and collateral constraints. Note that the planner does not set prices and instead takes the optimal pricing functions as given. However, the planner internalizes how its consumption and borrowing decisions affect all general equilibrium prices. As such, the planner behaves like a strategic player and not competitively as the households do in the previous section. Therefore, the equilibrium price of nontradable goods (p^N) and bonds (q, Q) will enter the SP problem as implementability constraints.[34] As before, the focus is on the Markov perfect stationary equilibrium. I assume that the planner cannot commit to future policy rules, including future defaulting and borrowing decisions. Consequently, it chooses current period allocations taking as given the strategies of future planners. Equilibrium is characterized by a fixed point of these policy rules. That is, the policy rules of future planners are consistent with the solutions to the optimization problem of the planner in the present period. Thus, the planner has no incentives to deviate from the future policy rules, thereby making these policies time consistent.[35]

The social planner's optimization problem consists of maximizing the utility of the households (1) subject to the credit constraint (2), the resource constraint (6), (7), and equilibrium prices (14),(15), and (16).The household budget constraint is automatically satisfied by Walras's law. Denote $\{\mathcal{L}^{SP\prime}$ and $\mathcal{B}'^{SP\prime}\}$ as the public and private borrowing decisions, respectively. Let $\boldsymbol{d}^{SP}$ be the default decisions of

[34] This formulation is equivalent to letting the planner make all borrowing decisions and transfer the proceeds to competitive households who make all consumption decisions taking prices as given.

[35] Other papers that follow using this approach to construct time-consistent public policies are Klein et al. (2008) and Bianchi and Mendoza (2018).

future planners that the current SP takes as given. The planning problem is[36]

$$W^{SP}(s,L,B) = \max_{d\in\{0,1\}} [1-d]W^{SP,R}(s,L,B) + dW^{SP,D}(s,B), \tag{17}$$

where the default value of the planner $W^{SP,D}(s,B)$ is

$$\begin{aligned}
W^{SP,D}\left(s,B\right) &= \max_{c^T,B'} u\left(c^T,y^N\right) - \phi(y^T) + \epsilon_{Def} + \beta\mathbb{E}_s\left[W^{SP}\left(s',0,B'\right)\right],\\
c^T + B(1-\pi) &= y^T + q^{SP}(s)B',\\
q^{SP}(s)B' &\leq \kappa\left(\frac{1-\omega}{\omega}\left(\frac{c^T}{y^N}\right)^{\eta+1} y^N + y^T\right),\\
q^{SP}(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r}.
\end{aligned} \tag{18}$$

and the value of the planner under repayment $W^{SP,R}(s,L,B)$ is

$$\begin{aligned}
W^{SP,R}(s,L,B) &= \max_{c^T,B',L'\in\Lambda} u\left(c^T,y^N\right) + \epsilon(L') + \beta\mathbb{E}_s[W^{SP}(s',L',B')],\\
c^T + B(1-\pi) + \delta L &= y^T + q^{SP}(s)B + Q^{SP}(s,L',B')[L'-(1-\delta)L],\\
q^{SP}(s)B' &\leq \kappa\left(\frac{1-\omega}{\omega}\left(\frac{c^T}{y^N}\right)^{\eta+1} y^N + y^T\right),\\
q^{SP}(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r}.
\end{aligned}$$

$$Q^{SP}(s,L',B') = \frac{1}{1+r}\times\mathbb{E}_s\left[\left[1-\boldsymbol{d}^{SP}(s',L',B')\right]\times\left[\delta+(1-\delta)Q^{SP}\left(s',\mathcal{L}^{SP\prime}(s',L',B'),\mathcal{B}^{SP\prime}(s',L',B')\right)\right]\right].$$

In contrast to the government in the baseline version, the planner directly controls the level of aggregate private borrowing B'. Like the government, the planner chooses aggregates. As a result, the planner's decisions consider their impact on all general equilibrium prices: the effect of the price of nontradables (14) on the private debt limit (2) and the equilibrium best response of the foreign lenders.

Definition 2. *A Markov stationary socially planned equilibrium is a set of value functions $\{W^{SP}, W^{SP,R}, W^{SP,D}\}$, policy functions for allocations $\{C^{SP,T}, C^{SP,N}, \mathcal{L}^{SP\prime}, \mathcal{B}^{SP\prime}\}$, and defaulting $\boldsymbol{d}^{SP}$, and pricing functions for public Q^{SP} and private q^{SP} debt, that solve (17) given conjecture future policies $\{C^{SP,T}, C^{SP,N}, \mathcal{L}^{SP\prime}, \boldsymbol{d}^{SP}\}$*

[36]For concision, the equilibrium price of nontradables (14) and the resource constraint of nontradables (6) are already incorporated in this formulation. The price of public bonds Q^{SP} is equated with the equilibrium best response of competitive risk-neutral lenders.

3.4 Decentralization with macroprudential policies

In this subsection, I consider another version of the model where the government gains access to state-contingent linear taxes on private borrowing. I show that the Markov competitive equilibrium allocation solves the socially planned problem presented in the previous subsection. The households' budget constraint (3) becomes

$$(1-\pi_t)b_t + c_t^T + p_t^N c_t^N = q_t(1-\tau_t)b_{t+1}, + y_t^T + p_t^N y^N + T_t,, \tag{19}$$

where τ_t is the tax rate on private borrowing. The introduction of taxes does not modify the credit constraint (2). As with all other government policies, taxes on private debt are taken as given by households. At the same time, the government can still tax the households using lump-sum transfers. The budget constraint (5) is now

$$T_t = (1-d_t)\Big[Q_t\big[L_{t+1} - (1-\delta)L_t\big] - \delta L_t\Big] + \tau_t q_t B_{t+1}. \tag{20}$$

Note that the government can still tax private debt and use lump-sum transfers while in default. Appendix A provides a complete recursive formulation and characterization of the decentralized equilibrium with taxes.

Proposition 1. *The socially planned equilibrium allocation can be decentralized with a state-contingent tax on debt that satisfies*

$$1 - \boldsymbol{\tau}(s, L, B) = \frac{\beta \mathbb{E}_s\Big[(1-\pi')\Big(u_T^{SP}(C^{SP,T}(s', L', B'), C^{SP,N}(s', L', B'))\Big)\Big] + \mu^{SP}(s, L, B)q^{SP}(s)}{q^{SP}(s)u_T(C^{SP,T}(s, L, B), y^N)}, \tag{21}$$

where μ^{SP} corresponds to the Lagrange multiplier associated with the credit constraint in the planner problem (17).

Proof: See Appendix B.

The proof is done in two steps. First, I show that the planning problem is equivalent to a relaxed version of the competitive equilibrium with taxes. Second, I show that solutions to the planning problem are sufficient to construct policies that satisfy the additional constraints of the competitive equilibrium problem with taxes.

3.5 Mechanism

This subsection explains the intuition behind the main mechanism of the model. For this purpose, I compare the intertemporal optimality conditions of the baseline and planner problems presented before. Consider the intertemporal optimality conditions of the households in the baseline problem (13),[37]

$$q(s)u_T(C^T(S_G)) = \beta\mathbb{E}_s[(1-\pi')u'_T(C^{T\prime}(S_G)] + \mu(S_G)q(s), \tag{22}$$

$$0 \leq \kappa(p^N(S_G)y^N + y^T) - q(s)\mathcal{B}'(S_G) \quad \text{with equality if } \mu(S_G) > 0, \tag{23}$$

where $u_T(.)$ is shorthand notation for $\frac{\partial u}{\partial c}\frac{\partial c}{\partial c^T}$, the marginal utility of consumption of tradables, and μ is the Lagrange multiplier on the credit constraint. Condition (22) is the household's Euler equation for private debt and (23) is the complementary slackness condition. If $\mu > 0$, the marginal utility benefits from increasing tradable consumption today exceed the expected marginal utility costs from borrowing one unit of private debt and repaying next period. The main difference between the baseline model and the planning problem is in their private borrowing decisions. Consequently, I compare the Euler equation of private bonds for each problem.[38] Using the same notation as before, the planner policies (SP) are[39]

$$\left(u_T^{SP}(C^{SP,T}) + \mu^{SP}\psi^{SP}\right)\left(q^{SP} + Q_{B'}^{SP}(\mathcal{L}^{SP\prime} - (1-\delta)L)\right) = \beta\mathbb{E}_s\left[(1-\pi')\left(u_T^{SP}(C^{SP,T\prime}) + \mu^{SP\prime}\psi^{SP\prime}\right)\right] + \mu^{SP}q^{SP}. \tag{24}$$

The prime notation denotes future states and the marginal utility of consumption, and the Lagrange multiplier are u_T^{SP} and mu^{SP}, respectively. In contrast to the baseline's condition (22), the planners' Euler equation includes the marginal effect on the collateral value of an additional unit of tradable consumption $\psi^{SP} = \kappa(1+\eta)\frac{(1-\omega)}{\omega}(\frac{C^{SP,T}}{y^N})^\eta$, public borrowing policies $\mathcal{L}^{SP\prime}$, and the marginal effect on the price of public bonds of an additional unit of tradable consumption $Q_{B'}^{SP}$. These terms capture the additional general equilibrium effects that the planner considers when deciding its level of private borrowing. While the first term is common in the Fisherian debt deflation literature, the latter two are encountered in the sovereign debt maturity management literature. I now briefly discuss the effect of each of them.

The term ψ^{SP} appears in Bianchi (2011). It captures that, relative to the households in the baseline model, the planner considers the marginal benefit of an extra unit of private borrowing on the current and future real exchange rate. First, additional borrowing increases the consumption of tradables and therefore the price of nontradables, which in turn relaxes the credit constraint ($\mu^{SP}\psi^{SP}$). Quantitatively, this effect is generally small, as numerically, it is usually that $\psi^{SP} < 1$. [40] Second, additional

[37] These expressions are obtained by assuming that the policy and value functions are differentiable and then applying the standard envelope theorem to the first-order conditions of the household problem and assuming that rational expectations hold.

[38] The complete characterization of the optimality conditions of the planning problem is discussed in Appendix A.

[39] As before, these first-order conditions are obtained by assuming differentiability and the standard envelope conditions. In addition, it also assumed that the equilibrium price of bonds is differentiable.

[40] If $\psi^{SP} > 1$, in some states this can instead lead to underborrowing and/or multiple equilibria. In all quantitative specifications considered in the paper, this case is never encountered. For specifications where this is violated, see Schmitt-Grohé and Uribe (2019). For other models of Fisherian deflation with underborrowing, see Benigno et al. (2013).

private borrowing decreases expected cash-in-hand next period, depressing the expected future price of nontradables ($\mu^{SP\prime}\psi^{SP\prime}$). Thus, additional borrowing increases the probability of facing a binding constraint next period. The planner internalizes this cost; the competitive households in the baseline model do not.[41] Consequently, the planner borrows less. This effect is quantitatively significant and the source of private overborrowing in the baseline model.

The terms $\mathcal{L}^{SP\prime}$ and $Q^{SP}_{B'}$ are seen in Arellano and Ramanarayanan (2012) and Hatchondo et al. (2016) in models where the government has access to public bonds of different maturities. The private bond discussed here has a short-term maturity and differs from the assets discussed in those papers in two ways. First, it is not directly controlled by the government in the baseline model but by the households. Second, it is not strategically defaultable and is instead subject to the collateral constraint. Nevertheless, some of the trade-offs described in those models apply here. Private borrowing increases the probability of default and also increases the expected issuances of public debt in case of repayment. Keeping all other things equal, an extra unit of private bonds decreases expected wealth next period. Mechanically, this increases the probability of sovereign default. Moreover, even in states of repayment, higher private debt increases the probability of a debt-financed bailout. As a result, in some states an extra unit of private debt is also associated with an increase in expected future public debt. As a consequence of these two effects, increasing private debt increases the premium paid on public debt. Since the planner optimally manages its issuances of both assets, it chooses a lower level of private debt to lower the interest paid on its public debt. Lenders internalize that the government in the baseline problem cannot guarantee this optimal portfolio in either the current or future periods. Consequently, lenders offer a worse price schedule to the government than to the counterfactual social planner. This bond schedule combined with more frequent use of public bailouts will quantitatively explain the difference in average spreads between the baseline and socially planned equilibria.

4 Quantitative analysis

In this section, I solve numerically the two versions of the model presented in the previous section. The baseline is solved using time iteration for the private equilibrium and value function iteration for the government problem. The socially planned economy can be solved by value function iteration. More details regarding the numerical solution methods are described in Appendices D and E.

4.1 Calibration

The baseline version of the model is calibrated using Spanish macroeconomic data from 1999 to 2011. One period in the model corresponds to one year in the data. I assume that Spain was at the ergodic distribution of the baseline version of the model during this period. The calibration consists

[41] Note that the decision to ignore this effect is rational from the individual household perspective. Each household is small and does not control aggregate borrowing. As a result, its borrowing cannot affect aggregate prices.

of selecting a set of parameters so that the ergodic distribution averages coincide with the relevant macroeconomic moments in the data.

The starting year is chosen to coincide with the creation of the Eurozone. Before this, most Spanish public debt was in domestic currency, and therefore its nominal value was subject to government choices. The end year of 2011 is chosen to keep out of sample the significant European policies introduced in 2012. Some of these policies conflict with some of the fundamental assumptions underlying the baseline version of the model. Although Spain had implemented countercyclical prudential policies for its domestic banking sector in 1999, up until 2011 there were no systematic controls on private international borrowing within the European Union.[42] This changed in June of 2012, when European heads of state proposed the creation of the Single Supervisory Mechanism (SSM) to supervise bank debt within the union. By 2014, the Bank of Spain had transferred a substantial portion of its supervisory powers to the SSM. In addition, in June of 2012, European leaders also agreed to allow the European Stability Mechanism to offer direct help to Spanish banks. Finally, one month later, in July 2012, then president of the European Central Bank (ECB) Mario Dragui famously signaled the commitment of the institution to do "whatever it takes to preserve the Euro." That statement was interpreted at the time as a commitment from the ECB to buy Eurozone public bonds from distressed countries.[43]

Given that the baseline version of the model assumes no restrictions in international private debt and that the last two mechanisms of supranational bailouts are not explicitly modeled, I restrict the sample to the year prior to their introduction. As a consequence of this assumption, in the next section, I will use the comparison between the model and the data responses to the large financial shock as an out-of-sample validation.

Functional forms. The utility function is of the constant relative risk aversion (CRRA) form on the composite CES good

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma} \quad \text{with } \sigma > 0.$$

The default utility cost is parameterized as follows:

$$\phi(y^T) = \max\{0, \phi_0 + \phi_1 \ln y^T\}.$$

As Arellano (2008) and Chatterjee and Eyigungor (2012) discuss, a nonlinear specification of the default costs allows the model to reproduce the mean and standard deviation of spreads in the data. In particular, I follow Bianchi et al. (2018) in specifying the default cost function in terms of utility.

[42] Saurina and Trucharte (2017) provide a detailed account of the history of banking regulation in Spain and how it adapted to the adoption of international accounting standards during this period. For an overview of the current provisions, see Mencia and Saurina Salas (2016).

[43] For a discussion of how beliefs can be crucial for sovereign default incentives, see Cole and Kehoe (2000), Conesa and Kehoe (2017), and Aguiar et al. (2020).

Table 1: Parameters estimated outside of the model

Description	Parameter	Value
Risk aversion	σ	2.0
Elasticity of substitution	$1/(1+\eta)$	.83
Share of tradables	ω	.39
Persistence of tradables	ρ^y	.75
Volatility of tradables	σ^y	.010
Mean private default rate	$\bar{\pi}$	.021
Persistence private default rate	ρ^π	.82
Volatility private default rates	σ^π	.33
Risk free interest rate	r	.027
Duration of long-term bonds	δ	.14

Note: The risk aversion and elasticity of substitution between tradables and nontradables are standard in the literature. The share of tradables is the average share of value added of agriculture, manufacturing, and tradable services on GDP. The risk-free rate is the average yield of one-year German Treasury bonds. The duration parameter is chosen to match the average bond duration of six years of Spanish bonds. The tradable income and private default shock parameters are estimated by fitting a first order autoregressive process on the logs of the tradable share of GDP and share of nonperforming gross loans, respectively. All public bonds and yield data are from 1999 to 2011, and the GDP and nonperforming loans process are estimated using the longest available series. The data source for bond yields and nonperforming loans is Bloomberg, and the sectoral GDP series are taken from Eurostat. For details, see data Appendix C.

Estimated parameters Table 1 shows the set of parameters that are estimated outside of the model. The risk aversion, σ, and elasticity of substitution between tradables and nontradables, $1/(1+\eta)$, are set at values frequently encountered in the literature.[44] To reduce the state space, the endowment of nontradables, y^N, is set to one. I assume that the endowment of tradables is drawn from a first-order lognormal autoregressive (AR(1)) process. I estimate this process using the cyclical component of linearly detrended tradable GDP for Spain. Since the focus is on fluctuations around the business cycle, I use the cyclical component of the linearly detrended share of tradable output[45]. The estimated values for persistence and volatility, respectively, are $\rho^y = .75$ and $\sigma^y = .01$. The recursive specification is

$$\ln y_t^T = \rho^y \ln y_{t-1}^T + \varepsilon_t^y \quad \text{with } \varepsilon_t^y \sim N(0, \sigma^y).$$

The value of ω is chosen to replicate the share of nontradable GDP in the data, which is 60%. [46] To compute the model counterpart of this object at the ergodic distribution, I use the mean value of external private and public liabilities of $\bar{b}$ and $\bar{L}$ at their targeted values.[47] The value of ω is then

[44] See for instance, Garcia-Cicco et al. (2010), and Bengui and Bianchi (2018).

[45] Details and sources in Appendix C.

[46] Tradable GDP is computed using the value-added shares of agriculture, manufacturing, and tradable services. More details can be found in Appendix C.

[47] In the baseline calibration described below, $\bar{b} = 0.42$ and $\frac{\delta}{1+\frac{1-\delta}{1+r}}\bar{L} = .14$

set so that $\frac{\bar{p}^N y^N}{\bar{p}^N y^N + y^T} = 0.60$ where $\bar{p}^N = \frac{1-\omega}{\omega}\frac{y^T - r\bar{b} - \delta r \bar{L}}{y^N}$. Since the average tradable and nontradable endowments are one, this yields $\omega = 0.39$.

Similarly, I assume that the exogenous share of private bonds defaulted on each period follows a log-normal AR(1) process. The parameters of this process are estimated using the gross share of nonperforming loans as a percentage of total loans.[48] The estimation yields an average private default rate $\bar{\pi} = 2.1\%$, a persistence parameter $\rho^\pi = .82$, and a volatility $\sigma^\pi = .33$. The recursive specification of the process is

$$\ln \pi_t = (1 - \rho^\pi)\bar{\pi} + \rho^\pi \ln \pi_{t-1} + \varepsilon_t^\pi \quad \text{with } \varepsilon_t^\pi \sim N(0, \sigma^\pi).$$

Two parameters affecting interest rates,r and δ, are estimated outside of the model. The risk-free interest rate is set to the average yield of the one-year German Treasury bill over the calibration period, $r = 2.7\%$. One-year bonds are chosen as a benchmark to reproduce the maturity of the short-term private bond in the model. The duration parameter δ is chosen so that average duration in the model corresponds to the average maturity of Spanish bonds in the data. Using Bank of Spain data, I find an average maturity of public debt of six years during the period of interest. This calculation is in line with previous estimates of Spanish maturity, such as Hatchondo et al. (2016) and Bianchi and Mondragon (2018). The Macaulay definition of duration of a bond given the coupon structure of the model is

$$D = \frac{1 + \bar{i_L}}{\delta + \bar{i_L}},$$

where $\bar{i_L}$ is the constant per-period yield delivered by a long-term bond held to maturity (forever) with no default.[49] The implied duration is then $\delta = .14$.

Table 2: Calibrated parameters

Description	Parameter	Value	Moment	Target	Model
Discount factor	β	.92	Mean total debt	.56	.56
Volatility taste shock	σ^ϵ	.020	Volatility total debt	.048	.050
Mean financial shock	$\bar{\kappa}$	.45	Mean private debt	.42	.42
Volatility financial shock	σ^κ	.020	Volatility private debt	.071	.058
Default cost	ϕ_0	.31	Mean spread	.0045	.0045
Default cost	ϕ_1	1.9	Volatility spread	.0061	.0061

Note: Total and private debt are computed using the international investment position presented in Section 2. Spreads correspond to the difference between the interest rate paid by Spanish six-year bonds and their German equivalents. All moments are computed using data from 1999 to 2011. For additional details, see Appendix C.

Calibrated parameters. Six parameters are calibrated to match six aggregate moments from the Spanish data. The calibrated parameters are the two constants in the default cost function ϕ_0 and ϕ_1,

[48] Details and sources are in Appendix C.

[49] In the baseline calibration, it corresponds to the targeted spread plus the risk-free rate, $\bar{i_L} = 3.1\%$.

the discount factor β, the standard deviation of the taste shocks σ^{ϵ}, and the constants determining the process of the financial shocks $\bar{\kappa}$ and σ^{κ}.[50] Table 2 shows a summary of all the targets and their model counterparts.

The parameters associated with the default costs ϕ_0 and ϕ_1 are measured in the data using the difference in returns between the average six-year Spanish bond and the average German bond of the same maturity. The targeted moments are the average and the standard deviation of this spread, and their model counterparts are the average and standard deviation of the spread paid by the long-term bond L_t. To compute the sovereign spread in the model that is implicit in a bond price Q, I use the definition of the constant per-period yield. Given the coupon structure, the yield satisfies

$$Q = \sum_{j=1}^{\infty} \delta \frac{(1-\delta)^{j-1}}{(1+\bar{i_L})^j}$$

The average targeted spread is 0.45% with a standard deviation of 0.47%, which implies values for the default cost parameters of $\phi_0 = .3$ and $\phi_1 = 1.9$. The targets are low when compared to the related literature since they are computed using data from 1999 to 2011. Other quantitative analyses of the sovereign debt crisis in Spain, such as Hatchondo et al. (2016) and Bianchi and Mondragon (2018), focus on spreads only from a latter period, 2011-2015, and, consequently, target a higher spread. This paper deviates from that by including in the calibration the years 1999-2007 when the interest rate spread of Spanish government debt was very close to zero. Since the aim of the paper is to study the link between the buildup of private debt during those years and the subsequent sovereign debt crisis, it is important for the model to simultaneously match both the years with zero spreads and the large spikes observed during the crisis. To achieve this, I calibrate the model so that the average spread at the ergodic distribution matches the near-zero environment, and in the next section, I see what the model predicts for the latter years when confronted with the shocks taken from the data.

The discount factor β and the volatility of the taste shocks σ^{ϵ} are selected to match the average and standard deviation of the total debt. To compute the model counterparts of these measures I first calculate the international positions of the public and private sectors. The stock of public debt as a percentage of output at time t in the model is calculated for our coupon structure as the present value of future payment obligations discounted at the risk-free rate, that is, $\frac{\delta}{1+\left(\frac{1-\delta}{1+r}\right)} \times \frac{L_t}{(p_t^N y_t^N + y_t^T)}$. By contrast, the international position of the private sector as a percentage of output at time t is simply $\frac{B_t}{(p_t^N y_t^N + y_t^T)}$. At the calibrated values, $\beta = .92$ and $\sigma^{\epsilon} = .02$.

Finally, since the buildup in private debt in the years leading up to the crisis is a motivating fact of the model, the last two targeted aggregated moments are the average and standard deviations of the private debt. Note that because of the symmetry in the evolution of private and public stocks, the

[50] The mean of the taste shocks is irrelevant for their quantitative properties and is selected to achieve numerical tractability. More details can be found in Appendix D.

volatility of the private and public positions is higher than the volatility of the total debt. It is therefore important that the model matches not only the aggregate positions but also some of its decomposition. I calibrate the process of financial shocks κ_t to match this. As with the other exogenous shocks in the model, I assume that the financial shock follows a first-order normal AR(1) process of the form

$$\kappa_{t+1} = (1-\rho^{\kappa})\bar{\kappa} + \rho^{\kappa}\kappa_t + \varepsilon_t^{\kappa} \quad \text{with } \varepsilon_t^{\kappa} \sim N(0, \sigma^{\kappa}).$$

For simplicity, I assume that the persistence parameter coincides with the persistence of tradable income $\rho^{\kappa} = \rho$, while the mean ($\bar{\kappa}$) and volatility parameters (σ^{κ}) are estimated within the model. The model successfully replicates the average debt of the private sector and a higher volatility for the private position relative to the aggregate. However, it fits less well the large standard deviation seen in the data. At the baseline calibration, $\bar{\kappa} = .45$ and $\sigma^{\kappa} = .02$.

4.2 Policy functions of private and public debt

To shed light on the workings of the model, this section shows an analysis of the policy functions for public and private debt accumulation. Both variables are functions of the exogenous shocks of the model and of the initial portfolio composition. To fix ideas, this section will first show how the accumulation of private and public debt varies with respect to the two main exogenous shocks, income and financial shocks. Then, I will show how both types of debt issuances vary with the endogenous states, the initial level of total debt and end-of-period public debt. Since the government acts first, the end-of-period private debt is a function of both the beginning of period debt of the country and the newly issued public debt. Considering the best response from the households, the government chooses the issuance of public debt optimally. For simplicity, the initial level of public debt has been set to zero in all the policy function plots, making all initial debt private. Nevertheless, all the implications follow through with a strictly positive level of initial public debt. Unless otherwise specified all debt levels are expressed as a share of mean output at the ergodic distribution.

Policy functions of private debt Figure 4 depicts the optimal private debt accumulation as a function of the income and financial shocks. Panel (a) shows end-of-period private debt as a function of the endowment of tradable shocks, for the mean value of κ and π_t and for two possible values of initial debt. Panel (b) plots end-of-period private debt as a function of the financial shock, for the mean value of y^T, again for two possible values of initial debt.

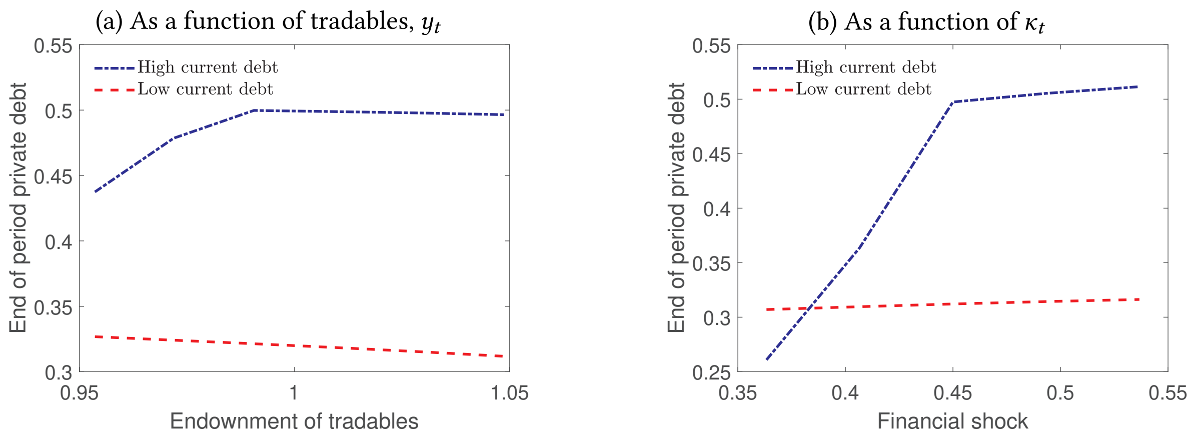

Figure 4: Policy function for private debt relative to the exogenous states

The figure shows that households' borrowing choices are most sensitive to the exogenous shocks when the households are facing a binding credit constraint. If the initial level of debt is low, represented by the dashed line in the plot, end-of-period private debt increases only slightly when income is low or the borrowing capacity is larger (smaller y^T or higher κ). However, if the current debt is high enough, households borrow up to their credit constraint. As a result, increases in the endowment of tradables or the value of the financial shock (higher y^T or higher κ) are met with equivalent increases in private borrowing.

Focusing now on the endogenous states, Figure 5 plots the law of motion of end-of-period private debt as a function of the initial level of debt, panel (a), and to next period public debt, panel (b). To help visualize the importance of the credit constraint, the total borrowing capacity of the private sector (debt limit) is plotted alongside the policy functions. In both panels, the exogenous shocks are kept constant. In the first panel, the level of end-of-period public debt is set at zero, and in the second panel, the starting level of debt is one standard deviation above the mean.

Panel (a) shows that for low levels of initial debt, the credit constraint does not bind, and end-of-period private debt increases with current total debt. The change in the sign of the slope of the policy function indicates the point at which the credit constraint is satisfied with equality. Beyond this point, higher levels of initial debt imply a lower level of tradable consumption. This in turn lowers the price of nontradables p^N and further restricts the borrowing capacity of the economy. This is therefore an illustration of the Fisherian debt deflation mechanism discussed in the previous section. As a result, similar policy functions can be seen Bianchi (2011) and Bianchi and Mendoza (2018).

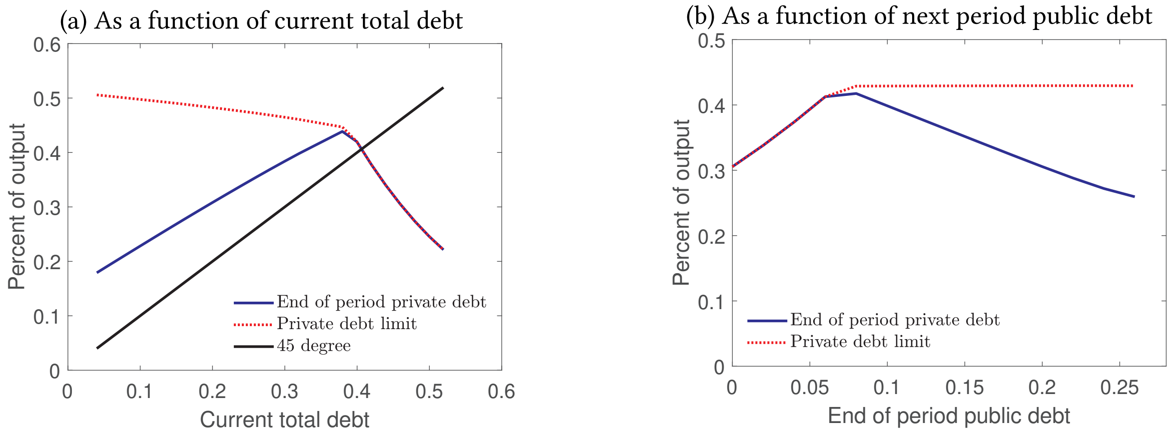

Figure 5: Policy function of private debt relative to the endogenous states

In contrast, panel (b) depicts the private sector response to the government's end-of-period debt and is novel to this paper. Low levels of end-of-period public debt imply a reduction in the fiscal transfer received by the household. At the plotted values, without substantial government assistance (above 8% of output), private borrowing will be constrained. Given the financial amplification mechanism described before, in this constrained area, higher government borrowing increases the consumption of tradables, the price of nontradables, the borrowing limit of the private sector, and private borrowing. This process comes to a halt once government assistance is large enough to ensure that the households will not borrow up to their limit. Further government borrowing continues to increase the transfer received by the households, but they now respond by borrowing less. For these states, private and public debt are substitutes.

Figure 6 shows the optimal public debt accumulation policy as a function of the income (panel (a)) and financial shocks (panel (b)). When initial debt is low, or when the endowment and the financial capacity κ are high, the optimal end-of-period debt remains mostly constant around a positive value. As in other models with multiple maturity assets, such as Arellano and Ramanarayanan (2012), long-term bonds provide rollover benefits relative to the short-term bonds. Long-term bonds provide more insurance against income fluctuation, which facilitates consumption smoothing. As a result, the government finds it optimal to always have a strictly positive level of public debt, even when the households are unconstrained. Since private and public debt are substitutes in these states, the government can issue debt at low spreads as long as total public debt remains low.

Policy functions of public debt The government considers the household's best responses when choosing the level of public borrowing. Since the choice of public debt is also affected by the taste shock drawn, I now plot the expected level of next period public debt conditional on repayment. All values are plotted as a share of output. I start by showing public debt as a function of the income and financial shocks and then show how it changes with initial debt.

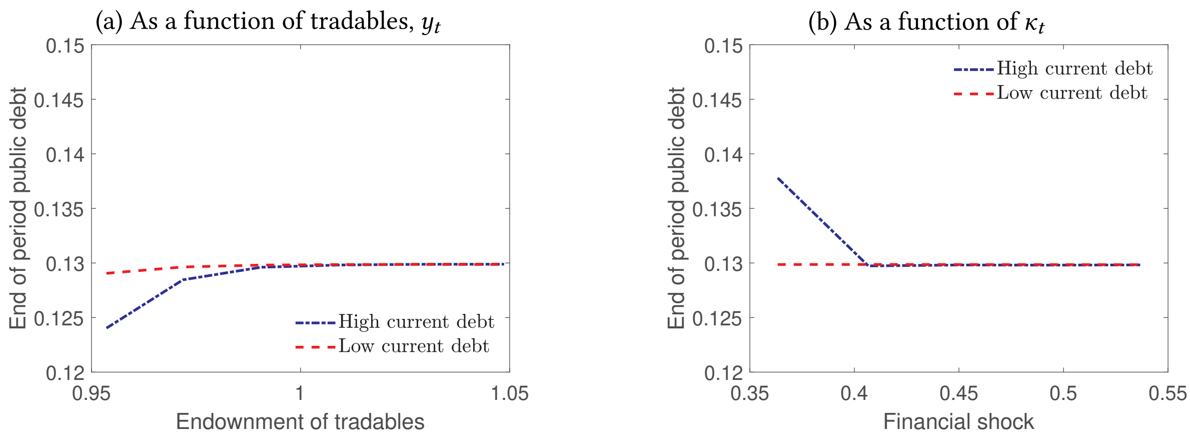

Figure 6: Policy function of public debt relative to the exogenous states

In contrast, when total debt is high, end-of period public debt varies differently with each type of exogenous shock. A low tradable endowment implies higher default risk and higher spreads, and therefore public borrowing decreases. Instead, an adverse financial shock (low κ) means that private borrowing is more likely constrained. Public debt in these cases has the twofold beneficial effect detailed in the previous section. Public debt allows for higher consumption when the households are constrained. This relaxes the credit constraint by depreciating the real exchange rate and allows for higher private borrowing. Thus, higher end-of-period public debt is desirable.

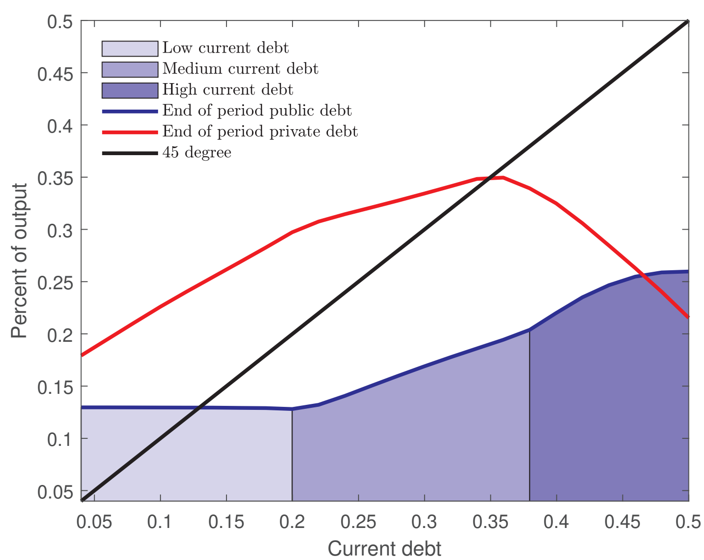

Figure 7: Expected end-of-period public and private debt as function of initial debt

Finally, Figure 7 shows the expected level of end-of-period public debt as a function of the current level of debt (blue line). To help visualize the situation of the households, the figure also shows the expected end-of-period private debt. All values are plotted as a share of output, and all exogenous shocks and the initial level of public debt are kept at constant values. Depending on the initial level of debt, three types of responses in terms of public debt are possible.

When the initial level of debt is low, issuances of public debt are kept relatively constant and low. Public debt is issued here because of its rollover benefits. Long-term debt allows the government to partially insure the households against transitory fluctuations in all exogenous shocks. Private debt is increasing in initial debt while public debt is almost constant. If the initial debt is large enough, however, the constraint for the private sector will bind if the government end-of-period debt is zero. At these medium levels of initial debt, households are not expected to face a credit constraint on average. The government is expected to transfer enough resources to the household so that the constraint will not bind. Consequently, private and public debt levels are increasing in the initial level of debt. The slope of private debt accumulation is smaller than in the previous case because households will be constrained in some states. Finally, if the initial level of debt is very high, it is never optimal to provide a large enough bailout that would prevent the households from facing a binding constraint. In these cases, issuances of public debt are at their highest. This is because in these states, public debt has a significant positive impact on the private borrowing capacity. The higher the initial level of debt, the more constrained the households are expected to end up, even after receiving transfers, and therefore the lower the level of end-of-period private debt.

Comparison with the socially planned economy A social planner who controls the issuance of both types of assets would have similar policy functions. In this subsection, we compare those policies to those presented in the baseline model discussed above.

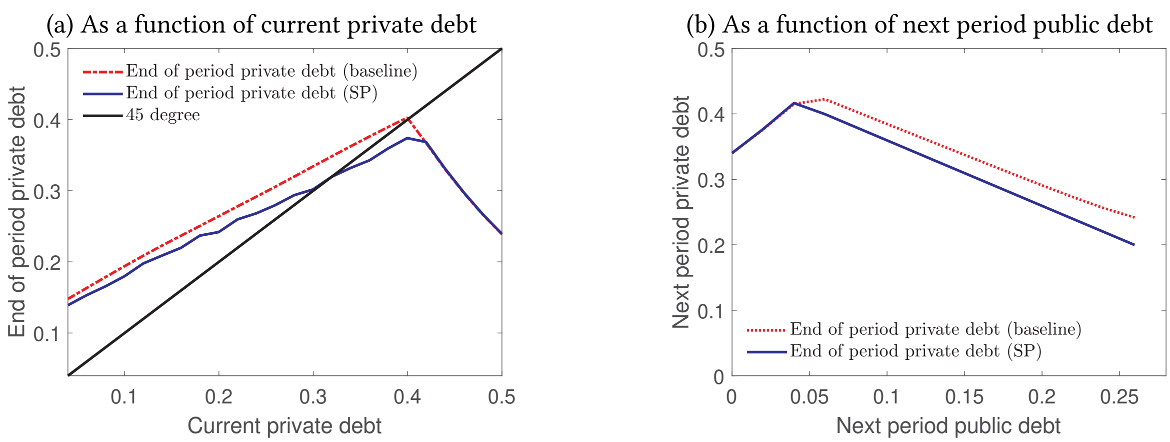

Figure 8: Policy function of private debt, baseline and SP

Figure 8 compares the evolution of end-of-period private debt in the baseline and socially planned economy as a function of the initial stock of private debt (panel (a)) and end-of-period public debt (panel (b)). In both panels, overborrowing in the baseline economy is present only when the constraint does not bind. When the constraint binds, private borrowing is pinned down by the resource constraints, and therefore there is no room for disagreement between the models. The sources of private overborrowing in both panels, however, are different. In the first panel, households overborrow for low levels of initial private debt because they do not internalize the marginal effect of their debt on the probability of facing a binding constraint next period. This figure is common to models of private overborrowing with a credit constraint that is increasing in the price of nontradables, such as Bianchi (2011) and Bianchi and Mendoza (2018). In contrast, the second panel is novel to this paper. Overborrowing is now caused by a smaller private borrowing response to government issuances of public debt. Unlike the planner, the households do not internalize that higher private debt increases the probability of sovereign default next period. Thus, individual households substitute less private debt for the same increase in public debt relative to the planner.

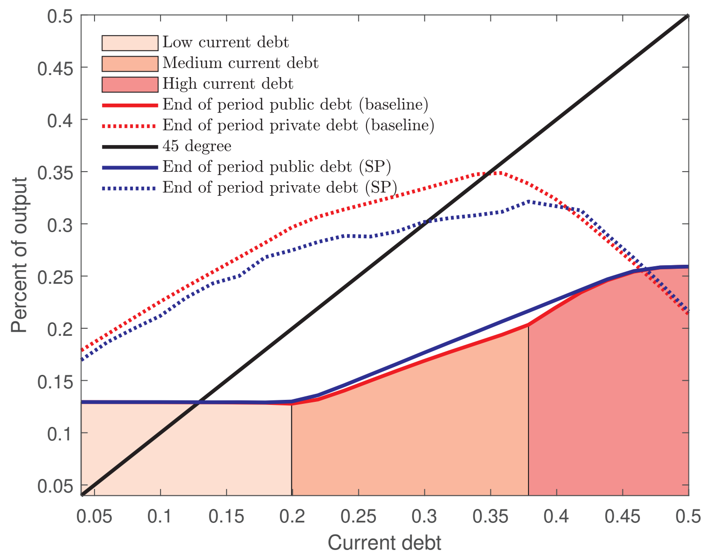

Figure 9: Expected end-of-period public and private debt as function of initial debt

Figure 9 compares the expected optimal level of public borrowing, conditional on repayment, in the baseline and socially planned economies as a function of the initial debt. As before, the households' private debt responses are plotted alongside the planners'. The figure also shows private overborrowing in the baseline model when the constraint does not bind. Public borrowing is higher in the planned economy when initial debt is small or medium. In these areas, the planner internalizes that it is approaching its borrowing capacity on the private bond and substitutes some of that borrowing with the public bond. The government in the decentralized economy would like to implement the same policy but does not control the issuances of the private bond. Correctly predicting that the household will not reduce private borrowing at the same rate as a planner would, the government decides to issue less public debt. The differences in public borrowing are, however, quantitatively smaller than the differences in private borrowing. As shown in the next section, when we compare the ergodic distributions, the small differences in public borrowing will not compensate for the fact that the baseline economy hits the credit constraint more often than the planned one. Consequently, the government must more frequently relieve the households by issuing public debt. When the constraint is expected to bind, the two economies mostly coincide.[51]

I also compare the evolution of the expected interest rate spreads paid on public debt in both economies conditional on repayment. Figure 10 plots the spreads as a function of the initial debt. The figure is computed at the same states as in Figure 9. The spreads peak when the debt enters the high debt zone. The shape of this plot shows that the interest rate spreads are mostly driven by the evolution of total end-of-period debt. Default is more likely in a more indebted economy. Up until the moment the constraint binds, both private and public debt are increasing with initial debt. Beyond this point, however, the private sector deleverages at a rate that outpaces the increase in public borrowing. As a result, total indebtedness decreases. This reduces the probability of default and the spread. In all cases, the spreads are higher in the baseline economy. This is the case even though Figure 10 shows that for medium or high levels of debt, the planner is expected to issue more public debt. The gap in interest rates exists because total debt is higher in the baseline economy as a result of household overborrowing. Anticipating this, foreign lenders demand a higher spread from the government.

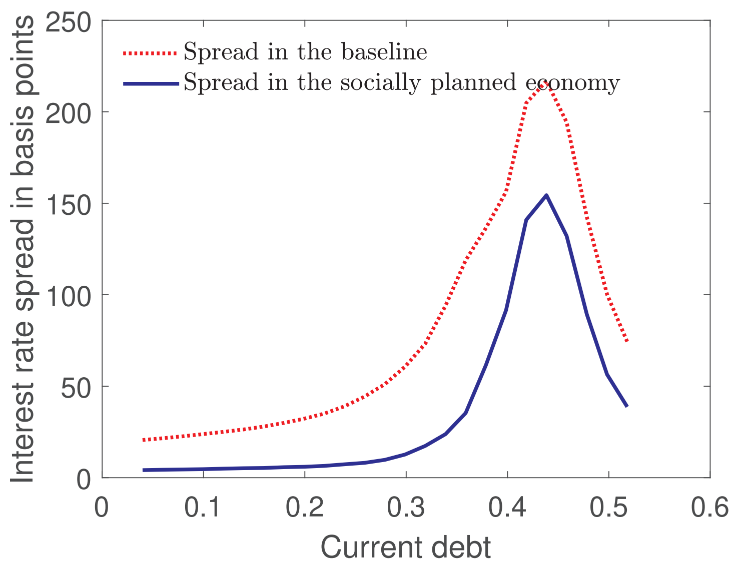

Figure 10: Expected spreads on public debt as function of initial debt

4.3 Untargeted business cycle properties

This subsection evaluates the model's quantitative performance by comparing untargeted moments from the data with moments from the model at the ergodic distribution. I compute the model's moments by simulating the exogenous processes for 10,000 periods and eliminating the first 500 observations. The moments from the data are computed with annual data for the sample period 1999-2017.

[51]The small amount of underborrowing in the baseline economy in this context is caused by fact that the planner faces a more favorable price schedule and therefore can relax the constraint a little bit more.

The longer sample period is chosen to avoid small sample bias. Similar results are obtained when restricting the sample to the period 1999-2011. In Table 3, real GDP is equated with output, and consumption corresponds to total final consumption expenditure and is measured in real terms. GDP and consumption data are detrended. The current account and trade balance are computed as a percentage of GDP. All data are from Eurostat, and additional descriptions of the sources can be found in Appendix C.

Table 3 compares the unconditional second moments in the Spanish data with their baseline model counterparts at the ergodic distribution. The model successfully captures the volatility of consumption, of the current account and of the trade balance, and overestimates the volatility of output. Nevertheless, the model correctly predicts that the volatility of output will exceed the volatility of consumption. This contrasts with traditional sovereign default models where the opposite is true.[52] This suggests that explicitly modelling international private debt is important to simultaneously achieve a volatility of consumption and net capital flows consistent with the Spanish data. Table 3 also computes correlations between output and the other business cycle statistics. The model correctly predicts the sign of all the correlations.

Table 3: Untargeted business cycle statistics

Statistic	Data	Calibration
Volatility		
Output	.032	.062
Consumption	.031	.037
Current account	.041	.046
Trade balance	.034	.040
Correlations		
Output - Consumption	.97	.99
Output - Current account	-.59	-.91
Output - Trade balance	-.54	-.94
Output - Spread on public debt	-.46	-.10
Public debt - Spread on public debt	.53	.28

Note: Output corresponds to real gross domestic product and consumption to real final consumption expenditure, and both series are detrended. Current account and trade balance are measured as a percentage of output. Public debt corresponds to the international investment position of the public sector. Spreads correspond to the difference between the interest rate paid by Spanish six-year bonds and their German equivalents. For additional details, see Appendix C.

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[52] Neumeyer and Perri (2005) find that consumption is more volatile than output in emerging economies whereas the opposite is true in advanced economies. Spain is listed by the International Monetary Fund (IMF) as an advanced economy.

5 Results: Quantitative implications of private overborrowing

This section details the main quantitative findings of the paper. The first subsection details the results obtained by comparing the baseline and regulated economies at their ergodic distributions. The second subsection details the quantitative exercises conducted to simulate the model dynamics during the Spanish Debt Crisis and the counterfactual in a socially planned economy.

5.1 Social planner and baseline economies at the ergodic

Table 4 presents the first set of quantitative results of the paper. The table shows the values first and second moments in the data and at the ergodic distribution of the baseline, and the socially planned economies. The baseline version of the model is calibrated to match the moments from the data; the socially planned economy is not. Instead, I use the calibrated parameters of the baseline to compute the ergodic distribution of the planned problem. The average private debt at the ergodic distribution for the social planner is 36% of output, whereas in the baseline case it is 41%. This difference of 5% of output is the estimate of the total amount of excessive private debt in Spain in the lead-up to the crisis. The table shows that the increase in private debt, in the baseline relative to the planner, is insufficient to explain the increase in overall indebtedness. The baseline economy accumulates on average more public debt, around 2% of output. The explanation for this can be seen in the bottom half of the table. In this part, I compute four measures of aggregate well-being for the baseline and planned economies, namely, the probability of a binding credit constraint, the probability of a financial crisis, the probability of a sovereign default, and a measure of welfare gains. The credit constraint binds more frequently under the baseline. As explained in the previous section, optimal government borrowing is higher when the constraint binds. As a result, average public debt is higher under the baseline because the government must respond more often to crisis. I define a financial crisis as an episode with a binding constraint and a contraction of more than one standard deviation below the mean of the current account of the private sector.[53] Under this definition, I find that excessive private borrowing increases the incidence of financial crises by 2.40 percentage points on average.

Furthermore, Table 4 compares the interest rate spreads paid on public debt relative to the risk-free rate. In the planned economy, spreads are on average an order of magnitude below their baseline counterparts. The reduction in the spread is caused both by the fact that the planner borrows less in general, and that it faces a binding constraint less often. The result is also consistent with the smaller average probability of sovereign default in the regulated economy relative to the baseline. Finally, Table 4 computes the welfare gains of moving from the baseline to the planned economy. The welfare gains are calculated as the proportional increase in consumption for all possible future states that would make the households indifferent between staying in the baseline and moving to the centralized

[53]Similar definitions are encountered in the related literature; see, for instance, Bianchi (2011) and Bengui and Bianchi (2018).

Table 4: Baseline and social planner aggregate moments at the ergodic distribution

Moment	Data	Baseline	Social planner
Total debt	.56	.56	.49
Private debt	.42	.42	.37
Mean spread	.0045	.0045	.00034
Volatility debt	.048	.050	.027
Volatility private debt	.071	.058	.071
Volatility spread	.0061	.0061	.00030
Probability of a binding constraint	-	.099	.024
Probability of a financial crisis	-	.025	.0010
Probability of default	-	.0046	.00030
Welfare gains	-	-	.0041

Note: All calibrated parameters are kept constant in the computation of the socially planned economy. A financial crisis is defined as a episode in which the credit constraint binds and the current account of the private sector contracts by more than one standard deviation below the mean. Welfare gains are calculated as the proportional increase in permanent consumption under the baseline. Debt levels in the data are calculated using the international investment positions. More details are explained in Appendix C.

equilibrium. This measure explicitly incorporates the cost of lower consumption in the transition to the ergodic distribution of the planned economy. Taking advantage of the homoscedasticity of the utility function, the expected welfare gains in state (s_0, L_0, B_0) are

$$\boldsymbol{\theta}(s_0, L_0, B_0) = \left(\frac{\boldsymbol{W}^{SP}(s_0, L_0, B_0) \times (1-\sigma) \times (1-\beta) + 1}{\boldsymbol{W}(s_0, L_0, B_0) \times (1-\sigma) \times (1-\beta) + 1} \right)^{\frac{1}{1-\sigma}} - 1. \tag{25}$$

On average at the ergodic state, households would need to receive a permanent increase of 0.36% in consumption to be indifferent between the two economies. These welfare gains are larger than the ones encountered in the literature. In Bianchi (2011), the welfare gains from correcting the overborrowing externality are around 0.13%. The welfare gains are larger in my model because optimal private debt management also decreases the probability of experiencing the deadweight losses of sovereign default.

5.2 Simulating the 2012 debt crisis

This section uses the data and the calibrated models to provide a model simulation of the events that unfolded in Spain between 2008 and 2015. To shed light on what optimal policies could have achieved, I also plot, alongside the baseline model and the data, the counterfactual dynamics of the socially planned economy. The idea is to feed into the model the exogenous shocks that affected Spain during this period and contrast the endogenous response in terms of debt and spreads of the baseline and socially planned models with their data counterparts. I conduct two exercises. In the first one, I feed into the model the three fundamental exogenous shocks: the income shock, the private default shock, and the financial shock. Public and private debt as well as the spread on public bonds are then allowed to respond endogenously to these shocks. In the second exercise, I impose as an additional restriction the evolution of public debt encountered in the data. I then compute the model-predicted dynamics of private debt and the interest rate spread. This second exercise therefore corresponds to the endogenous response of the baseline and socially planned economies when fixing fiscal policies to the data.

In both exercises, the exogenous income shock, y_t, is taken directly from the Spanish tradable GDP data. Similarly, the share of private bonds defaulted on, π_t matches exactly the data on gross nonperforming loans during this period. The taste shocks, ϵ_t, are all set to zero in the first exercise and selected to perfectly match the evolution of the public debt in the second. The financial shock, κ_t, is always unobserved in the data. To circumvent this problem, I apply the particle filter method proposed by Bocola and Dovis (2019) to my model. Additional details about the particle filter method can be found in appendix F; here I present a summary of the methodology.

The baseline model defines a nonlinear state-space system A

$$\begin{aligned}
\boldsymbol{Y}_t &= g(\boldsymbol{S}_t) + e_t, \\
\boldsymbol{S}_t &= f(\boldsymbol{S}_{t-1}, \varepsilon_t),
\end{aligned}$$

where $\boldsymbol{S}_t = [L_t, B_t, y^T_{t-1}, \pi_{t-1}, \kappa_{t-1}]$ is the state vector and ε_t the vector collecting all the innovations in the three structural exogenous shocks. The vector of observables, $\boldsymbol{Y}_t$, includes average private and public debt as a share of GDP, detrended tradable output, the share of nonperforming loans, and the interest rate spreads on public bonds.[54] The vector e_t represents uncorrelated Gaussian measurement errors and is equal to the difference between the data aggregates $\boldsymbol{Y}_t$ and their model counterparts $g(\boldsymbol{S}_t)$. The functions $g(\cdot)$ and $f(\cdot)$ come from the calibrated numerical solutions of the baseline model. The realizations of the state vector are estimated by applying the particle filter to this system of equations and data from 2008 to 2015. The process yields a path of financial shocks and a set of initial endogenous states. I then feed these shocks into the social planner policy functions $f^{SP}(\cdot)$ to generate the allocations of debts and spreads that would have emerged under counterfactual optimal policies. Note that the social planner functions are not used to estimate the system and are only used expost to generate counterfactuals. Finally, I also construct the implied tax on borrowing, necessary to implement the planner allocations in a competitive equilibrium.

[54] As in the calibration, I use the linearly detrended cyclical component of tradable output. Public debt is initialized at zero, and initial private debt is adjusted to match the composition of total debt in the data.

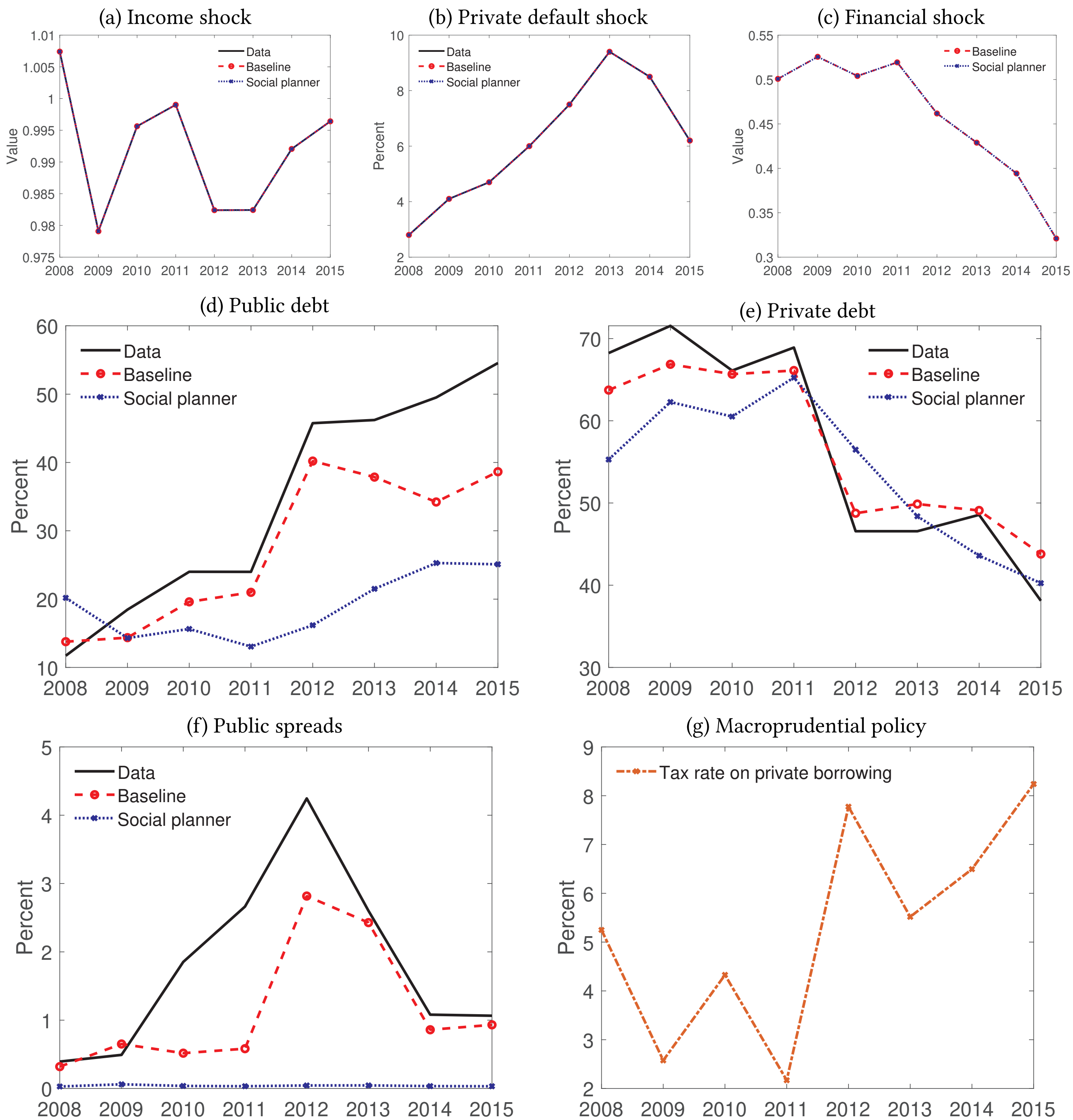

Figure 11: Evolution of debt, taxes, spreads, and exogenous shock, 2008–2015: data and models

Note: Model simulations are obtained by feeding into the model observed income shocks, nonperforming loans, and the most likely path of financial shocks from the particle filter. Public debt, private debt and spreads are the particle-filtered weighted averages. Both debt series are expressed as a percentage of output, while nonperforming loans are expressed as a percentage of gross loans. Taxes and interest rate spreads are expressed in percentages. Data sources can be found in Appendix C, while details on the particle filter can be found in Appendix F.

In the first exercise, I assume that only tradable output and nonperforming private loans are observed with no error. This leaves three observable variables not perfectly fitted in $\boldsymbol{Y}_t$: public debt, private debt, and spreads. To match them, there are three stochastic variables in $\boldsymbol{S}_t$, namely, B_t, L_t, and κ_t. By setting the variance of all measurement errors to 1% of their sample variance, I compute the filtered path of these three stochastic variables that is consistent with the data. Figure 11 summarizes the results of this exercise.

The baseline model, plotted in dashed red lines, captures the main events of the crisis. In particular, the magnitude of the 2012 public bailout, around 12% of GDP, is financed by an equivalent increase in public debt. This leads to an increase in the interest rate spread on public bonds of around 3%, equivalent to 80% of the increase observed in the data. The baseline model is less successful at tracking the evolution of public debt after 2012, predicting a lower indebtedness than what is observed in the data. Similarly, the interest rate spread increase in the model before 2012 is below its data counterparts. Two observations could partially explain these discrepancies. First, while the model captures some of the fluctuations in the external conditions for borrowing via the financial shock, it may be the case that this shock is not enough to fully replicate the uncertainty around government bonds of Eurozone countries during the worst years of the Greek debt crisis. Second, there is no model counterpart to the Mario Draghi speech of June 2012 that can replicate its effect on public interest spreads. Accordingly, the model expects less public debt than the data to replicate the drop in spreads observed in the 2013-2015 period. All things considered, the baseline model predicts a pattern of public debt, private debt, and spreads that is consistent with the data and validates the approach of the paper.

Having validated the positive model, I now turn to the normative counterfactual. In contrast to the baseline case, the socially planned economy is predicting a smooth transition from private liabilities to public debt. Instead of a large bailout in 2012, the planner deleverages in the private bond in three years. The dynamics allow the planner to maintain the interest rate spread near zero throughout the period and halves the size of the 2012 bailout to around 10% of GDP. Note that with the exception of 2012, private debt is lower in the planned economy in all years. The government could have implemented this with a macroprudential tax on private borrowing that is on average 5% during this period. Similarly, public debt in the socially planned economy is significantly below the levels observed in the data for most of the period, and importantly, even after the bailouts take place.

In the first exercise, the 2012 spike in the spread of public debt would have been completely avoided if a planner had managed public and private borrowing optimally. To disentangle how much of the difference is caused by lower public borrowing and how much is caused by excessive private debt, a second counterfactual exercise is conducted. Taking advantage of the probabilistic framework of the model, I can select the taste shocks ϵ_t such that the path of public debt coincides exactly with the one observed in the data in both the baseline and planned economies. The particle filter is then applied to back out the implied financial shock and the filtered endogenous evolution of private debt and the sovereign spread. As before, I then feed into the model this sequence of exogenous shocks to the planner policy functions to compute the counterfactual private debt, and spreads. Finally, I use the planner's policies to compute the optimal taxes on borrowing that could have decentralize these dynamics. The results of the second exercise are presented in Figure 12.

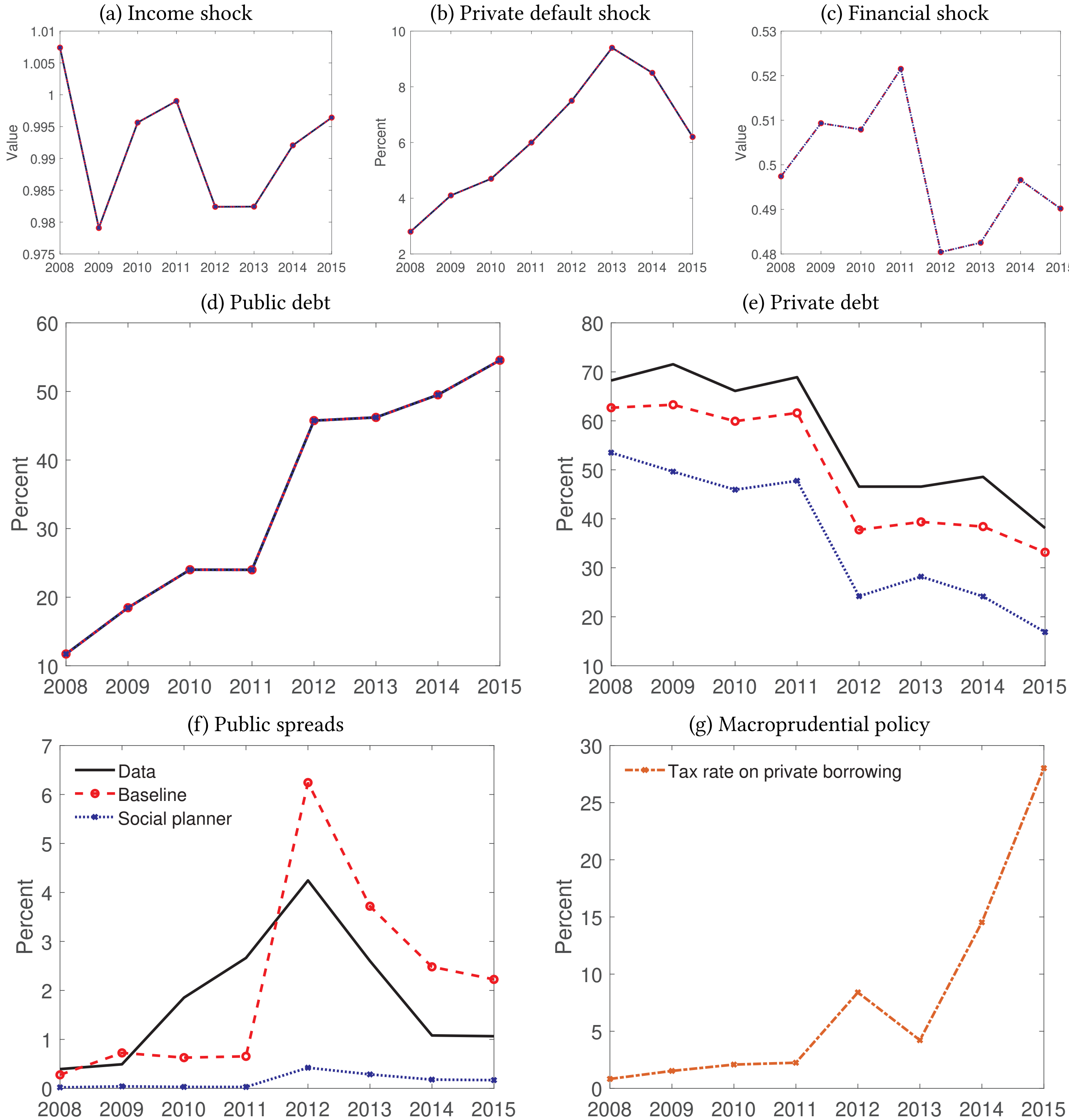

Figure 12: Evolution of debt, taxes, spreads, and exogenous shocks, 2008–2015: data and models

Note: Model simulations are obtained by feeding into the model observed income shocks, nonperforming loans, and taste shocks to match exactly the evolution of public debt. The most likely path of financial shocks is computed using the particle filter. Private debt and spreads are filtered weighted averages. Both debt series are expressed as a percentage of output, while nonperforming loans are expressed as a percentage of gross loans. Taxes and interest rate spreads are expressed in percentages. Data sources can be found in Appendix C, while details on the particle filter can be found in Appendix F.

The model once again predicts a drop in private debt of 20% of GDP, close in magnitude to the one observed in the data. Overall, private debt is around 5% below what is observed in the data for most of the period. The spread on public debt increases from close to zero in 2008, peaks in 2012, and then falls from 2013 onward. The magnitude of the increase between 2008 and 2012 is not the same in the baseline and the data, however, the model experiences a larger rise in 2012. The small mismatch in private debt and the larger spread are both consequences of the requirement to fit public debt exactly in this exercise. Nevertheless, the baseline model can still replicate the patterns of interest.

Finally, I compare the evolution of the data and the socially planned economy. Private indebtedness in the planned economy is still lower than in the baseline and the data. In this exercise, the data on the evolution of public debt impose that the main bailout takes place in 2012. As a result, the public spread in the planned economy also peaks in 2012. The peak value is .4%, or 3.8 percentage points below the spread observed in the Spanish data. This is the lower bound estimate of the increase in the severity of the sovereign debt crisis caused by excessive private debt. It should be restated here that this estimate is obtained while keeping the paths of public debt at their data values. The reduction in the spread is therefore not a consequence of lower public borrowing but of the only other endogenous factor, private debt. In the planned economy, the lenders internalize that the regulator will pair the increase in high public debt with high taxes on private debt, which is 8% on average during the period. This leads to a reduction in private debt and thus reduces the probability of a sovereign default in the future.

6 Conclusions

This paper develops a theory that is quantitatively consistent with the evolution of debt and spreads in Spain that culminated in the 2012 sovereign debt crisis. The theory presented here is also consistent with the business cycle statistics observed in the data during this time period.

The model focuses on the interaction between systemic externalities in private credit and sovereign default. The combination of competitive private households whose borrowing is constrained to a fraction of the market value of their current income and a benevolent government capable of assisting them with public funds creates a pathway from financial to sovereign debt crises. The process begins with a buildup of private debt when financial conditions are favorable. During this time, public debt remains low and the government faces low spreads. As the private sector accumulates more debt, a financial crisis becomes more likely. Eventually an adverse shock materializes, and the households face a tight borrowing limit. In the model, I allow for a crisis to be triggered by the following exogenous factors: slowdowns in output, increases in private default, and shocks to international financial markets. Confronted with an imminent and painful private deleveraging, the government responds with fiscal transfers financed by new issuances of public debt. Bailouts have a multiplicative positive effect in this context. A positive transfer causes an appreciation in the value of collateral, and through this channel increases the borrowing capacity of the private sector. As a result, bailouts allow credit-constrained households to issue more private debt and further increase consumption. Unfortunately, these gains come at the expense of raising the specter of a sovereign default. In all cases, the spreads paid on government debt increase, and in some particularly adverse circumstances, default materializes.

The paper also contributes to the literature by quantifying the level of excessive private borrowing and its impact. I estimate that in the lead-up to the crisis, excessive private debt in Spain was equivalent to 5% of GDP. As a result, the annual probability of experiencing a financial crisis was 2.4 p.p. above the socially desirable level. Finally, I estimate that private overborrowing raised the interest rate spread on public bonds by at least 3.8 p.p. at the peak of the sovereign debt crisis in 2012. As secondary findings, I calculate that private overborrowing raises the annual probability of a sovereign default from 0.03% to 0.46%. I demonstrate that optimal borrowing policies could have been implemented by pairing public debt management with state-dependent taxes on private borrowing. I estimate an average tax rate of 5% during the crisis years. Finally, I find that the welfare gains of implementing optimal borrowing policies would have been equivalent to an increase of 0.41% in aggregate consumption. Several interesting avenues for future research remain open. It would be fruitful to investigate the quantitative consequences of introducinc moral hazard into the motivations for private overborrowing. Alternatively, one could explore how budgetary covenants or other fiscal limits could simultaneously deal with the incentives for bailouts and with public debt dilution, as in Hatchondo et al. (2016) and Aguiar and Amador (2018). A final extension would be to investigate how a monetary response to private overborrowing would interact with the fiscal response presented here.

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Appendices

A Recursive competitive problem with taxes

For the representative household, the aggregate state of the economy includes the exogenous aggregate shocks denoted by $s = \{y^T, y^N, \kappa, \pi, \epsilon\}$, the initial level of government debt L, the initial level of aggregate private debt B, and the initial level of its own debt b. Following the same notation than in the body of the paper I denote $S = (s, L, B)$ the state space of the economy before government actions. Similarly, let $S_G = (S, d, L', \tau)$ denote the state space after government actions. Note that now that state includes the choice of taxes.

As before, households take as given the price of non-tradables $p^{N\tau}(S_G)$, the equilibrium price of price bonds $q^\tau(s)$, and government's current and future decisions regarding default $\boldsymbol{d}^{tau}$, public debt $\mathcal{L}^\tau$, and taxes $\boldsymbol{\tau}$. They also know the functions associates with these choices, the lump-sum transfer $\mathcal{T}^\tau$ and the preference shock $\mathcal{D}^\tau$. Finally, they also have a perceived law of motion of aggregate private debt $\mathcal{B}^{\tau\prime}$. The household's optimization problem in recursive form is:

$$V^\tau(S_G, b) = \max_{b', c^T, c^N} u(c(c^T, c^N)) + D + \beta \mathbb{E}_s [V^\tau(S'_G, b')] \tag{26}$$

$$\begin{aligned} &\text{subject to} \\ c^T + p^{N,\tau}(S_G)c^N + (1-\pi)b &= y^T + p^{N,\tau}(S_G)y^N + q^\tau(s)(1-\tau)b' + T, \\ q^\tau(s)b' &\leq \kappa[p^{N,\tau}(S_G)y^N + y^T], \\ T &= \mathcal{T}^\tau(S_G), \\ D &= \mathcal{D}^\tau(S_G), \\ B' &= \mathcal{B}^{\tau\prime}(S_G), \\ L' &= \mathcal{L}^{\tau\prime}(S_G) \\ \tau &= \boldsymbol{\tau}(S_G), \\ \text{And } S'_G &= (s', L', B', \boldsymbol{d}^\tau(s', L', B'), \mathcal{L}^{\tau\prime}(s', L', B'), \boldsymbol{\tau}(s', L', B')). \end{aligned}$$

Using the same notation than in the baseline case for the aggregate laws of motion of the private sector are $\mathcal{B}^{\tau\prime}(S_G)$, and $\{C^{i,\tau}(S_G)\}_{i=T,N}$, and public bond pricing $\boldsymbol{Q}^\tau(s, L', B')$ function. The government's problem is:

$$W^\tau(S) = \max_{d \in \{0,1\}} [1-d]W^{R.\tau}(S) + dW^{D,\tau}(S) \tag{27}$$

In case of default, $S_G = (S, 1, 0, \tau)$ and $W^{D,\tau}(S)$ is given by:

$$W^{D,\tau}(S) = \max_{\tau} u\Big(C^T, C^N\Big) + \epsilon^{Def} - \phi(y^T) + \beta \mathbb{E}_s\Big[W^\tau\big(s', 0, B'(S_G)\big)\Big] \tag{28}$$

$$\text{subject to}$$

$$\begin{aligned}
C^{T,\tau}(S_G) + (1-\pi)B &= y^T + q^\tau(s)(1-\tau)B' + T \\
C^{\mathcal{N},\tau}(S_G) &= y^N \\
T &= q^\tau(s)\tau B' \\
D &= \epsilon^{Def} - \phi(y^T) \\
B' &= \mathcal{B}^{\tau\prime}(S_G)
\end{aligned}$$

Note that transfers can still be strictly positive in default since the government transfers the proceeds to of the private debt tax to the households. In case of repayment, $S_G = (S, 0, L', \tau)$ and the value is:

$$W^{R,\tau}(S) = \max_{\tau, L' \in \Lambda} u\big(C^{T,\tau}, C^{N,\tau}\big) + \epsilon(L') + \beta \mathbb{E}_s[W^\tau(s', L', B')] \tag{29}$$

$$\text{subject to}$$

$$\begin{aligned}
C^{\mathcal{T},\tau}(S_G) + (1-\pi)B &= y^T + q^\tau(s)(1-\tau)B' + T, \\
C^{\mathcal{N},\tau}(S_G) &= y^N, \\
T &= Q^\tau(s, L', \tau, B')[L' - (1-\delta)L] - \delta L + q^\tau(s)\tau B', \\
D &= \epsilon(L'), \\
B' &= \mathcal{B}^{\tau\prime}(S_G)
\end{aligned}$$

The solution to the government's problem yields decision rules for default $\boldsymbol{d}^\tau(S)$, public borrowing $\mathcal{L}^{\tau\prime}(S)$, and taxes $\boldsymbol{\tau}(S)$. The transfers $\mathcal{T}^\tau(S_G)$ and preference shifter $D^\tau(S_G)$ are also pinned down by these decisions. The solution to the problem of competitive risk neutral foreign lenders yields the bond price schedule for private debt:

$$q^\tau(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r}, \tag{30}$$

and for public debt:

$$Q^{\tau}(s,L',B') = \frac{1}{1+r} \times \mathbb{E}_s\Bigg[\Big[1-d'\Big] \times \Big[\delta + (1-\delta)Q^{\tau}\Big(s',L'',B''\Big)\Big]\Bigg], \tag{31}$$

Where:

$$B'' = \mathcal{B}^{\tau\prime}(s',L',B'),$$
$$L'' = \mathcal{L}^{\tau\prime}(s',L',B'),$$
$$d' = \boldsymbol{d}^{\tau}(s',L',B')$$

Definition 3. *A Markov regulated competitive equilibrium with taxes is defined by, a set of value functions $\{V^{\tau}, W^{\tau}, W^{R,\tau}, W^{D,\tau}\}$, policy functions for the private sector $\{\hat{b}^{\tau\prime}, \hat{c}^{T,\tau}, \hat{c}^{N,\tau}\}$, policy functions for the public sector $\{\boldsymbol{d}^{\tau}, \mathcal{L}^{\tau\prime}, \boldsymbol{\tau}\}$, a pricing function for nontradable goods $p^{N,\tau}$, pricing functions for public debt Q^{τ} and private debt q^{τ}, and perceived laws of motion $\{\mathcal{B}^{\tau\prime}, C^{T,\tau}, C^{N,\tau}\}$ such that*

1. *Given prices $\{p^{N,\tau}, q^{\tau}\}$, government policies $\{\boldsymbol{d}^{\tau}, \mathcal{L}^{\tau\prime}, \boldsymbol{\tau}\}$, and perceived law of motion $\mathcal{B}^{\tau\prime}$, the private policy functions $\{\hat{b}^{\tau\prime}, \hat{c}^{T,\tau}, \hat{c}^{N,\tau}\}$ and value function V solve the household's problem (26)*

2. *Given bond prices $\{Q^{\tau}, q\}$ and aggregate laws of motion $\{\tilde{B}^{\tau\prime}, \tilde{C}^{T,\tau}, \tilde{C}^{N,\tau}\}$, the public policy functions $\{\boldsymbol{d}^{\tau}, \mathcal{L}^{\tau\prime}, \boldsymbol{\tau}\}$ and value functions W^{τ}, $W^{R,\tau}$, and $W^{D,\tau}$, solve the Bellman equations (27)–(29)*

3. *Households' rational expectations: perceived laws of motion are consistent with the actual laws of motion $\{\mathcal{B}'(S_G) = \hat{b}^{\tau\prime}(S_G,B), C^{T,\tau}(S) = \hat{c}^{T,\tau}(S_G,B), C^{N,\tau}(S_G) = \hat{c}^{N,\tau}S_G,B)\}$*

4. *The private bond price function $q^{\tau}(s)$ satisfies (30)*

5. *Given public $\{\boldsymbol{d}^{\tau}, \mathcal{L}^{\tau\prime}, \boldsymbol{\tau}\}$, and private $\{\mathcal{B}^{\tau\prime}\}$, policies the public bond price $Q^{\tau}(s, \mathcal{L}^{\tau}(S)', \mathcal{B}^{\tau}(S_G)')$ satisfies (31)*

6. *Goods market clear:*

$$C^{N,\tau}(S_G) = y^N$$
$$C^{T,\tau}(S_G) + (1-\pi)B = y^T + q^{\tau}(s)\mathcal{B}^{\tau\prime}(S_G) + \Big\{1-\boldsymbol{d}^{\tau}(S)\Big\}\Big\{Q^{\tau}(s,\mathcal{L}^{\tau}(S)',\mathcal{B}^{\tau}(S_G)')\Big[\mathcal{L}^{\tau\prime}(S) - (1-\delta)L\Big] - \delta L\Big\}$$

Similarly to the baseline model the optimality conditions of the households problem are:

$$q^{\tau}(s)(1-\boldsymbol{\tau}(S)u_T(C^{T,\tau}(S_G)) = \beta\mathbb{E}_s[(1-\pi')u_T(C^{T,\tau\prime}(S_G))] + \mu^{\tau}(S_G)q^{\tau}(s),$$

$$p^{N,\tau}(S_G) = \frac{1-\omega}{\omega}\left(\frac{C^{T,\tau}(S_G)}{y^N}\right)^{\eta+1},$$

$$0 \leq \kappa(p^{N,\tau}(S_G)y^N + y^T) - q^{\tau}(s)\mathcal{B}^{\tau\prime}(S_G) \quad \text{with equality if } \mu^{\tau}(S_G) > 0,$$

where μ^{τ} is the Lagrange multiplier associated with the credit constraint.

B Proof of proposition 1

This is a proof by construction. We will show that the recursive equilibrium with taxes can be written as a government problem that coincides with the planning problem (17). Start from the recursive competitive equilibrium problem with taxes described in Appendix B.

The problem with taxes is equivalent to the recursive problem of a government given that chooses allocations for the current period while taking future policies and prices as given. Denote these policies $\{\boldsymbol{d}^{\tau}(S), \mathcal{L}^{\tau\prime}(S), \boldsymbol{\tau}(S), C^{T,\tau}(S_G), C^{N,\tau}(S_G), \mathcal{B}^{\tau\prime}(S_G)\}$. This government maximizes utility considering the optimal responses of households and lenders. This is equivalent to let the government choose all policies using the Kuhn-Tucker conditions of households and lenders as constraints. The problem is therefore:

$$W^{\tau}(S) = \max_{d\in\{0,1\}} [1-d]W^{R,\tau}(S) + dW^{D,\tau}(S),$$

Let $S' = (S', B', L')$ the default value $W^{D,\tau}(S)$ is:

$$W^{D,\tau}(S) = \max_{c^T, c^N, B', \tau, \mu} u\left(c^T, c^N\right) - \phi(y^T) + \epsilon_{Def} + \beta \mathbb{E}_s\left[W^{\tau}(S')\right]$$

$$\text{subject to}$$

$$\begin{aligned}
c^T + B(1-\pi) &= y^T + q^{\tau}(s)B', \\
c^N &= y^N, \\
q^{\tau}(s)B' &\leq \kappa\left(p^{N,\tau}c^N + y^T\right), \\
q^{\tau}(s)(1-\tau)u_T(c^T, c^N) &= \beta E_s[(1-\pi')u_T(C^{T,\tau}, C^{N,\tau}(S', \boldsymbol{d}^{\tau}(S'), \mathcal{L}^{\tau\prime}(S'), \boldsymbol{\tau}(S'))] + \mu q^{\tau}(s) \\
p^{N,\tau} &= \frac{1-\omega}{\omega}\left(\frac{c^T}{c^N}\right)^{1+\eta} \\
(\kappa(p^{N,\tau}c^N + y^T) - q^{\tau}(s)B')\mu &= 0 \\
\mu &\geq 0 \\
q^{\tau}(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r}
\end{aligned}$$

The value under repayment $W^{R,\tau}(S)$ is:

$$
\begin{aligned}
W^{R,\tau}(S) &= \max_{c^T,c^N,B',\tau,\mu,L'\in\Lambda} u\left(c^T,c^N\right) + \epsilon(L') + \beta\mathbb{E}_s[W^\tau(S')] \\
&\text{subject to} \\
c^T + B(1-\pi) + \delta L &= y^T + q^\tau(s)B + Q^\tau(s,L',B')[L'-(1-\delta)L], \\
q^\tau(s)B' &\leq \kappa\left(p^{N,\tau}c^N + y^T\right), \\
q^\tau(s)(1-\tau)u_T(c^T,c^N) &= \beta\mathbb{E}_s[(1-\pi')u_T(C^{T,\tau},C^{N,\tau}(S',\boldsymbol{d}^\tau(S'),\mathcal{L}^{\tau\prime}(S'),\boldsymbol{\tau}(S'))] + \mu q^\tau(s) \\
p^{N,\tau} &= \frac{1-\omega}{\omega}\left(\frac{c^T}{c^N}\right)^{1+\eta} \\
(\kappa(p^{N,\tau}c^N + y^T) - q^\tau(s)B')\mu &= 0 \\
\mu &\geq 0 \\
q^\tau(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r}
\end{aligned}
$$

$$
Q^\tau(s,L',B') = \frac{1}{1+r}\times\mathbb{E}_s\left[\left[1-\boldsymbol{d}^\tau(S')\right]\times\left[\delta+(1-\delta)Q^\tau\left(s',\mathcal{L}^{\tau\prime}(S'),\mathcal{B}^{\tau\prime}(S',\boldsymbol{d}^\tau(S'),\mathcal{L}^{\tau\prime}(S'),\boldsymbol{\tau}(S'))\right)\right]\right]
$$

Substituting in the resource constraint for non tradables, and the intratemporal conditions that problem can be simplified to:

$$
W^\tau(S) = \max_{d\in\{0,1\}} [1-d]W^{R,\tau}(S) + dW^{D,\tau}(S), \tag{32}
$$

where default value $W^{D,\tau}(S)$ is:

$$
\begin{aligned}
W^{D,\tau}(S) &= \max_{c^T,B',\tau,\mu} u\left(c^T,y^N\right) - \phi(y^T) + \epsilon_{Def} + \beta\mathbb{E}_s\left[W^\tau(S')\right] \\
c^T + B(1-\pi) &= y^T + q^\tau(s)B', \\
q^\tau(s)B' &\leq \kappa\left(\frac{1-\omega}{\omega}\left(\frac{c^T}{y^N}\right)^{1+\eta}y^N + y^T\right) \\
q^\tau(s) &= \frac{\mathbb{E}_s[1-\pi']}{1+r} \\
q^\tau(s)(1-\tau)u_T(c^T,y^N) &= \beta\mathbb{E}_s[(1-\pi')u_T(C^{T,\tau},C^{N,\tau})] + \mu q^\tau(s) \\
0 &= \left[\kappa\left(\frac{1-\omega}{\omega}\left(\frac{c^T}{y^N}\right)^{1+\eta}y^N + y^T\right) - q^\tau(s)B'\right]\mu \\
\mu &\geq 0
\end{aligned}
$$

and value under repayment $W^{R,\tau}(S')$ is:

$$W^{R,\tau}(S') = \max_{c^T,B',\tau,\mu,L'\in\Lambda} u\big(c^T, y^N\big) + \epsilon(L') + \beta\mathbb{E}_s[W^\tau(S')]$$

$$c^T + B(1-\pi) + \delta L = y^T + q^\tau(s)B + Q^\tau(s,L',B')[L' - (1-\delta)L] \tag{33}$$

$$q^\tau(s)B' \leq \kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta}y^N + y^T\Big) \tag{34}$$

$$q^\tau(s) = \frac{\mathbb{E}_s[1-\pi']}{1+r} \tag{35}$$

$$Q^\tau(s,L',B') = \frac{1}{1+r}\times\mathbb{E}_s\bigg[\Big[1-\boldsymbol{d}^\tau\Big]\times\Big[\delta + (1-\delta)Q^\tau\Big(s', \mathcal{L}^{\tau\prime}, \mathcal{B}^{\tau\prime}\Big)\Big]\bigg] \tag{36}$$

$$q^\tau(s)(1-\tau)u_T(c^T,y^N) = \beta\mathbb{E}_s[(1-\pi')u_T(C^{T,\tau}, C^{N,\tau})] + \mu q^\tau(s) \tag{37}$$

$$0 = \bigg[\kappa\Big(\frac{1-\omega}{\omega}\big(\frac{c^T}{y^N}\big)^{1+\eta}y^N + y^T\Big) - q^\tau(s)B'\bigg]\mu \tag{38}$$

$$\mu \geq 0 \tag{39}$$

In this formulation it is apparent that the social planner problem (17) is a relaxed version of problem (32). In problem (32) the government must satisfy three additional constraints (37)–(39) and has access to two additional instruments μ and τ. Crucially, both μ and τ only appear in problem (32) in constraints (37)–(39). As such, problem (17) will be equivalent to problem (32) if we can use the solutions of (17) to construct two functions $\mu(s,L,B)$ and $\tau(s,L,B)$ that satisfy (37)–(39).

Let $\{C^{SP,T}(s,L,B), C^{SP,N}(s,L,B), \mathcal{L}^{SP\prime}(s,L,B), \mathcal{B}^{SP\prime}(s,L,B), \boldsymbol{d}^{SP}(s,L,B), Q^{SP}, q^{SP}(s)\}$ be a solution of problem (17). Additionally let $\mu^{SP}(s,L,B) \geq 0$ be the multiplier on the collateral constraint of the planner problem (17). μ^{SP} corresponds to the shadow value of relaxing the collateral constraint from the planner's perspective. This multiplier is different from μ which corresponds to the shadow value of relaxing the collateral constraint for individual households, and is a variable chosen by the government in (32). The complementary slackness condition of the social planner problem (17) is:

$$0 = \bigg[\kappa\Big(\frac{1-\omega}{\omega}\big(\frac{C^{SP,T}(s,L,B)}{y^N}\big)^{1+\eta}y^N + y^T\Big) - q^{SP}(s)\mathcal{B}^{SP\prime}(s,L,B),'\bigg]\mu^{SP}(s,L,B). \tag{40}$$

As such by setting:

$$\mu(s,B,L) = \mu^{SP}(s,L,B)$$

$$1-\boldsymbol{\tau}(s,L,B) = \frac{\beta\mathbb{E}_s\Big[(1-\pi')\Big(u_T^{SP}(C^{SP,T}(S'), C^{SP,N}(S'))\Big)\Big] + \mu^{SP}(s,L,B)q^{SP}(s)}{q^{SP}(s)u_T(C^{SP,T}(s,L,B), y^N)},$$

We can see that (37)–(39) are satisfied and therefore the two problems are equivalent.

C Data Appendix

Gross Domestic Product (GDP): *Eurostat March 2019, National accounts aggregates by industry up to NACE A*64, nama_10_a64,-*. Corresponds to Total gross value added in all NACE activities. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2015.

Non-tradable share of GDP: *Eurostat March 2019, National accounts aggregates by industry up to NACE A*64, nama_10_a64*. Corresponds to the share of total value added produced in the following industries: public administration, wholesale and retail, construction, and real state. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2015.

Tradable share of GDP: *Eurostat March 2019, National accounts aggregates by industry up to NACE A*64, nama_10_a64*. Corresponds to the complement of nontradable valued added as a share of total value added. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2015.

Private debt: *Chapter 17 of the statistical bulletin of March 2019, Banco de España (2019), table 21c "Breakdown by institutional sector"*. Corresponds to the inverse of the net international investment position of Spanish monetary financial institutions (excluding the Bank of Spain) and other resident sectors. The data series used are 3273771 and 3273777. Data is annualized from quarterly data from March 1999 to December 2015 and is in millions of Euros. In the calibration we use data only from 1999 to 2011,

Public debt: *Chapter 17 of the statistical bulletin of March 2019, Banco de España (2019), table 21c "Breakdown by institutional sector"*. Corresponds to the inverse of the net international investment position of the Bank of Spain and all public administrations. The data series used are 2386960 and 3273774. Data is annualized from quarterly data from March 1999 to December 2015 and is in millions of Euros. In the calibration we use data only from 1999 to 2011,

Total debt: *Chapter 17 of the statistical bulletin of March 2019, Banco de España (2019), table 21c "Breakdown by institutional sector"*. Corresponds to the inverse of the net international investment position of Spain and is calculated as the consolidation of private and public positions. Data is annualized from quarterly data from March 1999 to December 2015 and is in millions of Euros. In the calibration we use data only from 1999 to 2011.

Risk free rate: *Bloomberg ticker GTDEM1Y Govt*, Corresponds to the average interest rate spread paid on 1 year German treasury bonds. Data is annualized from quarterly data from March 1999 to December 2011.

Spread on public bonds: *Bloomberg tickers GTESP6YR Govt and GTDEM6Y Govt*, Corresponds to the difference between average interest rate paid on 6 year Spanish treasury bonds and 6 year German treasury bonds. Data is annualized from quarterly data from March 1999 to December 2015. In the calibration we use data only from 1999 to 2011.

Average Maturity: *Table 5 from the Bank of Spain's economic bulletin Alloza et al. (2019), of March 2019*, Average maturity of the stock of public debt for Spain in years. Annual data from 1999 to 2011.

Nonperforming loans: *Bloomberg ticker BLTLWESP Index*, Nonperforming loans as a share of total gross loans. Annual data from 1999 to 2015.

Consumption: *Eurostat , GDP and main components (output, expenditure and income) nama_10_gdp*. Corresponds to final consumption expenditure. The data is in chain linked volumes (2010) millions of Euros. Frequency is annual from 1999 to 2017.

Current Account: *Eurostat, Balance of Payments BOP_GDP6_Q, table TIPSBP11*. Corresponds to current account as a percent of GDP. Definitions are based on the IMF's Sixth Balance of Payments Manual (BPM6). The data is unadjusted. Frequency is annual from 1999 to 2017.

Trade Balance: *Eurostat, Balance of Payments BOP_GDP6_Q, table TIPSBP11*. Corresponds to the balance of trade on goods and services as a percent of GDP. Definitions are based on the IMF's Sixth Balance of Payments Manual (BPM6). The data is unadjusted. Frequency is annual from 1999 to 2017.

D Solution Method: The Government's ex-ante problem

Following the approach of Dvorkin et al. (Forthcoming), I can re-write the government's Bellman equations before the ϵ shocks are realized. From an ex-ante point of view, the shocks ϵ make the default and borrowing decisions stochastic. By taking expectations over these shocks, the decisions can be viewed as probabilistic. If we view the previously defined equilibrium as a game between the private and public sector each period, the ϵ shocks allow the government to play mixed strategies. This makes the computation of this problem using value function iteration possible. We follow this approach to write (8) from a an ex-ante perspective. That is when all the aggregate states have realized except the ϵ. For this we summarize all other exogenous state variables in $z = (y^T, y^N, \kappa, \pi)$. As mentioned in the main text we assume that L' is a finite and bounded grid with $\mathcal{J}$ elements. Denote by $F(\epsilon)$ the joint cumulative density function of the taste shocks and by $f(\epsilon)$ its joint density function. To simplify notation in what follows, the following operator to denotes the expectation of any function

$Z(\epsilon)$ with respect to all the elements of ,

$$\boldsymbol{Z} = \mathbb{E}_{\epsilon} Z(\boldsymbol{\epsilon}) = \int_{\epsilon_1}\int_{\epsilon_2} ... \int_{\epsilon_{\mathcal{J}+1}} Z(\epsilon_1, .., \epsilon_{\mathcal{J}+1}) f(\epsilon_1, .., \epsilon_{\mathcal{J}+1}) d\epsilon_1, ..d\epsilon_{\mathcal{J}+1} \tag{41}$$

Given this notation we have that:

$$\boldsymbol{W}(z, L, B) = E_{\boldsymbol{\epsilon}}[W(s, L, B)]$$

$$\boldsymbol{W}(z, L, B) = E_{\boldsymbol{\epsilon}}\Big[\max\left\{W^R(s, L, B); W^D(s, B)\right\}\Big]$$

$$\boldsymbol{W}(z, L, B) = E_{\boldsymbol{\epsilon}}\Big[\max\Big\{\max_{L'\in\Lambda}\{u(C(s, L, B)) + \epsilon(L') + \beta\mathbb{E}_{z'|z}\boldsymbol{W}(z', L', \mathcal{B}'(s, L, B))\};$$

$$u(C(s, 0, B)) - \phi(y^T) + \epsilon^{Def} + \beta\mathbb{E}_{z'|z}\boldsymbol{W}(z', 0, \mathcal{B}'(s, 0, B))\Big\}\Big]$$

Subject to the resource constraints:

$$C^T(s, L, B) = y^T + q(s)\mathcal{B}'(s, L, B) - (1-\pi)B + Q(s, L', B')[L' - (1-\delta)\mathcal{B}'(s, L, B)] - \delta\mathcal{B}'(s, L, B)$$

$$C^N(s, L, B) = y^N$$

Furthermore, if its convenient to define the following expected utility objects:

$$\Upsilon_{L,L'}(z, B) = u(C(s, L, B)) + \beta\mathbb{E}_{z'|z}\boldsymbol{W}(z', L, \mathcal{B}'(s, L, B))$$

$$\Upsilon_{def}(z, B) = u(C(s, 0, B)) - \phi(y^T) + \beta\mathbb{E}_{z'|z}\boldsymbol{W}(z', 0, \mathcal{B}'(s, 0, B))$$

Lemma 2. *Suppose that the $\boldsymbol{\epsilon}$ shocks follow a multivariate generalized extreme value distribution with parameters $\{m, v, p\}$ and are i.i,d over time. Where v is the scale parameter and p is the shape parameter and is set to 1. m corresponds to the location parameter and is set to $-v\gamma$ where γ is the Euler constant. Suppose that public debt L is on a grid with $\mathcal{J}$ points. Then the ex-ante value function of the government's recursive problem can be re-written as*

$$\boldsymbol{W}(z, L, B) = \Upsilon_{def} + v\log\left[1 + \left(\sum_{L'\in\Lambda}\exp\left(-\frac{\Upsilon_{def} - \Upsilon_{L,L'}}{pv}\right)\right)^p\right] \tag{42}$$

Additionally given this distributional assumptions there are closed form solutions for the ex-ante probability of default and borrowing policy functions conditional on repayment.

Proof. Given our distributional assumptions

$$F(\boldsymbol{\epsilon}) = \exp\left[-\left(\sum_{j=1}^{\mathcal{J}}\exp\left(-\frac{\epsilon_j - m}{v}\right)\right) - \exp\left(-\frac{\epsilon_{\mathcal{J}+1} - m}{v}\right)\right] \tag{43}$$

For $j \in [\![0, \mathcal{J}+1]\!]$ we denote by $F_j(\boldsymbol{\epsilon}) = \frac{\partial F(\boldsymbol{\epsilon})}{\partial \epsilon_j}$, the marginal with respect to element j^{th} element of $\boldsymbol{\epsilon}$.

$$F_j(\boldsymbol{\epsilon}) = \begin{cases} \frac{1}{v} \exp\left[- \left(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\epsilon_j - m}{v}) - \exp(-\frac{\epsilon^{def} - m}{v}) \right) \right] & \exp(-\frac{\epsilon_j - m}{v}) \qquad for\ j = 1..\mathcal{J} \\ \frac{1}{v} \exp\left[- \left(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\epsilon_j - m}{v}) - \exp(-\frac{\epsilon^{def} - m}{v}) \right) \right] & \exp(-\frac{\epsilon^{def} - m}{v}) \qquad for\ j = \mathcal{J}+1 \end{cases}$$

Using this notation ant the dropping the states (z, B) from the previously defined $\Upsilon_{L,L'}(z, B)$ functions we can compute the ex-ante policy functions of the government in close form solutions. Let the probability of default be $\boldsymbol{d}(z, L, B) = \mathbb{E}_{\boldsymbol{\epsilon}} \boldsymbol{d}(z, L, B, \boldsymbol{\epsilon})$. Note that:

$$\begin{aligned} \boldsymbol{d}(z, L, B) &= \int_{-\infty}^{\infty} F_{\mathcal{J}+1}(\Upsilon_{def} + \epsilon^{def} - \Upsilon_1, ..., \Upsilon_{def} + \epsilon^{def} - \Upsilon_{def}) d\epsilon^{def} \qquad (44) \\ &= \int_{-\infty}^{\infty} \frac{1}{v} \exp\left[- \left(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} + \epsilon^{def} - \Upsilon_j - m}{v}) - \exp(-\frac{\epsilon^{def} - m}{v}) \right) \right] \exp(-\frac{\epsilon^{def} - m}{v}) d\epsilon^{def} \\ &= \int_{-\infty}^{\infty} \frac{1}{v} \exp\left[- \exp(-\frac{\epsilon^{def} - m}{v}) \left(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} - \Upsilon_j}{v}) + 1 \right) \right] \exp(-\frac{\epsilon^{def} - m}{v}) d\epsilon^{def} \end{aligned}$$

Define $\exp(\phi_{def}) = 1 + \sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} - \Upsilon_h}{v})$. We can use this to rewrite (44) as:

$$\begin{aligned} \boldsymbol{d}(z, L, B) &= \int_{-\infty}^{\infty} \frac{1}{v} \exp\left[- \exp(-\frac{\epsilon^{def} - m}{v}) \exp(\phi_{def}) \right] \exp(-\frac{\epsilon^{def} - m}{v}) d\epsilon^{def} \\ &= \frac{1}{v \exp(\phi_{def})} \underbrace{\int_{-\infty}^{\infty} \exp\left[- \exp(-\frac{\epsilon^{def} - m - v\phi_{def}}{v}) \right] \exp(-\frac{\epsilon^{def} - m - v\phi_{def}}{v}) d\epsilon^{def}}_{=v} \\ &= \frac{1}{1 + \left(\sum_{L' \in \Lambda} \exp\left(- \frac{\Upsilon_{def} - \Upsilon_{L,L'}}{v} \right) \right)} \qquad (45) \end{aligned}$$

Where the last equivalence uses the fact that the PDF of the generalized extreme distribution integrates to 1. Similarly, conditional on repayment, the random component $\boldsymbol{\epsilon}$ make the public borrowing decisions random from an ex-ante perspective. Given a set of current aggregate states relevant for the government, it is useful to introduce the probability of choosing an amount of public debt L' *conditional on not defaulting* as:

$$\boldsymbol{G}_{z,L,B}(L') = \mathbb{P}_{\boldsymbol{\epsilon}}\big(L' | d(z, L, B, \boldsymbol{\epsilon}) = 0\big)$$

Using the same notation as before we have that for the L' that is the j^{th} element of Λ:

$$
\begin{aligned}
\boldsymbol{G}_{z,L,B}(L') =& \frac{1}{1-\boldsymbol{d}(z,L,B)} \int_{-\infty}^{\infty} F_j(\Upsilon_j + \epsilon^j - \Upsilon_1, ..., \Upsilon_j + \epsilon^j - \Upsilon_{def}) d\epsilon^j \\
=& \frac{1}{(1-\boldsymbol{d}(z,L,B))v} \times \\
& \int_{-\infty}^{\infty} \exp\left[-\exp(-\frac{\epsilon^j - m}{v}) \left(\sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_j - \Upsilon_h}{v}) + \exp(-\frac{\Upsilon_j - \Upsilon_{def}}{v}) \right) \right] \exp(-\frac{\epsilon^j - m}{v}) d\epsilon^j
\end{aligned}
$$

Defining $\exp(\phi_j) = \exp(-\frac{\Upsilon_j - \Upsilon_{def}}{v}) + \sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_j - \Upsilon_h}{v})$, we can simplify:

$$
\begin{aligned}
\boldsymbol{G}_{z,L,B}(L') =& \frac{1}{(1-\boldsymbol{d}(z,L,B))v} \int_{-\infty}^{\infty} \exp\left[-\exp(-\frac{\epsilon^j - m}{v}) \exp(\phi_j) \right] \exp(-\frac{\epsilon^j - m}{v}) d\epsilon^j \\
=& \frac{1}{(1-\boldsymbol{d}(z,L,B))v \exp(\phi_j)} \underbrace{\int_{-\infty}^{\infty} \exp\left[-\exp(-\frac{\epsilon^j - m - v\phi_j}{v}) \right] \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) d\epsilon^j}_{=v} \\
=& \frac{1}{(1-\boldsymbol{d}(z,L,B)) \exp(\phi_j)}
\end{aligned}
$$

Finally this can be further simplified to:

$$
\begin{aligned}
\boldsymbol{G}_{z,L,B}(L') =& \frac{1}{(1-\boldsymbol{d}(z,L,B))} \times \frac{\exp(\Upsilon_j/v)}{\exp(\Upsilon_{def}/v) + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})} \\
=& \frac{\exp(\Upsilon_{def}/v) + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})}{\sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})} \frac{\exp(\Upsilon_j/v)}{\exp(\Upsilon_{def}/v) + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h}{v})} \\
=& \frac{1}{\sum_{H \in \Lambda} \exp\left(\frac{\Upsilon_{L,H} - \Upsilon_{L,L'}}{v} \right)}
\end{aligned} \tag{46}
$$

Finally the value $\boldsymbol{W}(z, L, B)$ is given by:

$$\boldsymbol{W}(z,L,B) = \sum_{j=1}^{\mathcal{J}+1} \int_{-\infty}^{\infty} (\Upsilon_j + \epsilon_j) F_j(\Upsilon_j + \epsilon^j - \Upsilon_1, ..., \Upsilon_j + \epsilon^j - \Upsilon_{def}) d\epsilon^j$$

$$= \sum_{j=1}^{\mathcal{J}} \int_{-\infty}^{\infty} \frac{\Upsilon_j + \epsilon_j}{v} \times$$

$$\exp\Big[- \exp(-\frac{\epsilon^j - m}{v}) \Big(\sum_{h=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_j - \Upsilon_h}{v}) + \exp(-\frac{\Upsilon_j - \Upsilon_{def}}{v}) \Big) \Big] \exp(-\frac{\epsilon^j - m}{v}) d\epsilon^j$$

$$+ \int_{-\infty}^{\infty} \frac{\Upsilon_{def} + \epsilon_{def}}{v} \times$$

$$\exp\Big[- \exp(-\frac{\epsilon^{def} - m}{v}) \Big(\sum_{j=1}^{\mathcal{J}} \exp(-\frac{\Upsilon_{def} - \Upsilon_j}{v}) + 1 \Big) \Big] \exp(-\frac{\epsilon^{def} - m}{v}) d\epsilon^{def}$$

$$= \sum_{j=1}^{\mathcal{J}} \exp(-\phi_j) \times$$

$$\Big[\Upsilon_j + m + v\phi_j + \underbrace{\int_{-\infty}^{\infty} (\frac{\epsilon_j - m - v\phi_j}{v}) \exp\big[- \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) \big] \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) d\epsilon^j \Big]}_{=v\gamma}$$

$$+ \exp(-\phi_{def}) \times$$

$$\Big[\Upsilon_{def} + m + v\phi_{def} + \underbrace{\int_{-\infty}^{\infty} (\frac{\epsilon^{def} - m - v\phi_{def}}{v}) \exp\big[- \exp(-\frac{\epsilon^{def} - m - v\phi_{def}}{v}) \big] \exp(-\frac{\epsilon^{def} - m - v\phi_{def}}{v}) d\epsilon^{def} \Big]}_{=v\gamma}$$

Where in the last equivalence we have used the fact that for all j:

$$\Upsilon_j + m + v\phi_j = \frac{(\Upsilon_j + m + v\phi_j) \int_{-\infty}^{\infty} \exp\big[- \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) \big] \exp(-\frac{\epsilon^j - m - v\phi_j}{v}) d\epsilon^j \Big]}{v}$$

The last step (underscored in the above equations) uses one of the integral properties of the Euler constant. We now use the fact we assumed the distribution of shocks to be mean zero, that is $m = -\gamma v$. Using the definition of ϕ_{def} one can see that:

$$\exp(-\phi_{def}) [\Upsilon_{def} + v\phi_{def}] = \frac{\Upsilon_{def} + v \log(1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v}))}{1 + \sum_{h=1}^{\mathcal{J}} \exp(\frac{\Upsilon_h - \Upsilon_{def}}{v})}$$

The value of the government is then given by:

$$\boldsymbol{W}(z,L,B)=\sum_{j=1}^{\mathcal{J}}\exp(-\phi_j)[\Upsilon_j+v\phi_j]+\exp(-\phi_{def})[\Upsilon_{def}+v\phi_{def}]$$

$$\boldsymbol{W}(z,L,B)=\sum_{j=1}^{\mathcal{J}}\frac{\Upsilon_j+v\log(\exp(-\frac{\Upsilon_j-\Upsilon_{def}}{v})+\sum_{h=1}^{\mathcal{J}}\exp(-\frac{\Upsilon_j-\Upsilon_h}{v}))}{\exp(-\frac{\Upsilon_j-\Upsilon_{def}}{v})+\sum_{h=1}^{\mathcal{J}}\exp(-\frac{\Upsilon_j-\Upsilon_h}{v})}+\exp(-\phi_{def})[\Upsilon_{def}+v\phi_{def}]$$

$$\boldsymbol{W}(z,L,B)=\sum_{j=1}^{\mathcal{J}}\frac{\Upsilon_j-\frac{v\Upsilon_j}{v}+v\log(\exp(\frac{\Upsilon_{def}}{v})+\sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v}))}{\exp(-\frac{\Upsilon_j}{v})(\exp(\frac{\Upsilon_{def}}{v})+\sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v}))}+\exp(-\phi_{def})[\Upsilon_{def}+v\phi_{def}]$$

$$\boldsymbol{W}(z,L,B)=\frac{v\log(\exp(\frac{\Upsilon_{def}}{v})+\sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v}))}{\exp(\frac{\Upsilon_{def}}{v})+\sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h}{v})}\sum_{j=1}^{\mathcal{J}}\exp(\frac{\Upsilon_j}{v})+\exp(-\phi_{def})[\Upsilon_{def}+v\phi_{def}]$$

$$\boldsymbol{W}(z,L,B)=\frac{\Upsilon_{def}+v\log(1+\sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v}))}{1+\sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v})}\sum_{j=1}^{\mathcal{J}}\exp(\frac{\Upsilon_j-\Upsilon_{def}}{v})+\exp(-\phi_{def})[\Upsilon_{def}+v\phi_{def}]$$

$$\boldsymbol{W}(z,L,B)=\left[\frac{\Upsilon_{def}+v\log(1+\sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v}))}{1+\sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v})}\right]\left[\sum_{j=1}^{\mathcal{J}}\exp(\frac{\Upsilon_j-\Upsilon_{def}}{v})+1\right]$$

$$\boldsymbol{W}(z,L,B)=\Upsilon_{def}+v\log(1+\sum_{h=1}^{\mathcal{J}}\exp(\frac{\Upsilon_h-\Upsilon_{def}}{v})) \tag{47}$$

To sum up the distributional assumptions allow us to obtain closed form solutions for the ex-ante value function (47), the policy functions for default (45), the public borrowing conditional on repayment (46), ∎

Note that the functions $\boldsymbol{G}_{z,L,B}(L')$ and $\boldsymbol{d}(z,L,B)$ are sufficient to express all government decisions. Using the fact that the shocks are i.i.d over time, and assuming a guess $\boldsymbol{Q}$ of next price schedule functions, we can use $\boldsymbol{G}_{z,L,B}(L')$ and $\boldsymbol{d}(z,L,B)$ to write the pricing equation of public bonds (16):

$$Q(z,L',B')=q(z)\mathbb{E}_{z'|z}\Bigg[\big[1-\boldsymbol{d}(z',L',B')\big]\big[\delta+(1-\delta)\sum_{L''\in\Lambda}\boldsymbol{Q}\big(z',L'',\mathcal{B}'(z',L',B')\big)\boldsymbol{G}_{z',L',B'}(L'')\big]\Bigg] \tag{48}$$

In the quantitative section we assume that the shocks are mean zero ($m=-\gamma v$). We also assume that the shape parameter p is one, therefore taste shocks are independent from each other within the period as well. The scale parameter v is calibrated to match the variance of public debt in the data.

E Numerical Solution

In this section we provide more detail about the solution methods we use to solve both the baseline and planner version of the model described in the main text. For both solutions methods we use the closed form ex-ante solutions of the government's problem described in detail in Appendix D.

Baseline. This version is solved in three steps. The first step solves the households problem while taking government policies and bond prices as given using time iteration method. The second step uses the implied policy functions of the private sector from the first step and the assumed bond schedules, and computes the closed form solutions that solve the government's ex-ante problem. Finally using private and public policy functions the schedule of private bonds is updated. Iterate until convergence in private en public policies.

- Construct a finite grid of initial public debt L and private debt B.
- Discretize the 3 exogenous shocks, income, financial shock and private default and its transition probability matrix using Tauchen approximation. Solve for the implied schedule of private bonds $q(\pi)$ using (15).
- Provide an initial guess of ex-ante policy functions for government default $\boldsymbol{d}(z, L, B)$, and borrowing probabilities conditional on repayment $\boldsymbol{G}(z, L, B, L')$.
- Provide an initial guess for the schedule of public bonds $\boldsymbol{Q}(z, L', B')$.
- Construct the implied transfer function $\boldsymbol{T}(z, B, L, L')$ using the government budget constraint (5).
- Taking all these functions as given find the optimal private borrowing $B'(z, L, B, L')$ and consumption decisions $C'(z, L, B, L')$ using the private sector Euler equation (22) to find the binding and non binding states.
- Given households optimal policies $B'(z, L, B, L')$, and $C'(z, L, B, L')$, and the guess schedule of public bonds $\boldsymbol{Q}(z, L', B')$, compute the ex-ante default and borrowing policy functions of the government using (45) and (46). Update the government policy functions.
- Compute the government ex-ante value function $W(z, L, B)$ using (47).
- Update the schedule of public bonds $\boldsymbol{Q}(z, L', B')$ using (48).
- Repeat until convergence in $W(z, L, B)$,$B'(z, L, B, L')$, and $C'(z, L, B, L')$, and $\boldsymbol{Q}(z, L', B')$ is achieved.

Social planner. This version is solved in three steps. The first step finds optimal private borrowing on a grid (*grid search method*) given an initial guess of public for each potential default and public borrowing decisions. The second step uses this optimal private borrowing policy and the assumed bond schedules to computes the closed form solutions for public borrowing and default and the value function. Finally using private and public borrowing policy functions the schedule of private bonds is updated. Iterate until convergence in private borrowing policies and the value function is achieved.

- Construct a finite grid of initial public debt L and private debt B.
- Discretize the 3 exogenous shocks, income, financial shock and private default and its transition probability matrix using Tauchen approximation. Solve for the implied schedule of private bonds $q(\pi)$ using (15).
- Construct a grid of potential private borrowing choices B'.
- Provide an initial guess of ex-ante policy functions for government default $\boldsymbol{d}^{SP}(z, L, B)$, and borrowing probabilities conditional on repayment $\boldsymbol{G}^{SP}(z, L, B, L')$.
- Provide an initial guess for the schedule of public bonds $\boldsymbol{Q}^{SP}(z, L', B')$.
- Taking all these functions as given find the optimal private borrowing $B^{SP\prime}(z, L, B, L')$ in the finite grid discarding all choices that violate the credit constraint (18) for each potential public borrowing and default decision.
- Given optimal private borrowig policy $B^{SP\prime}(z, L, B, L')$ and the guess schedule of public bonds $\boldsymbol{Q}^{SP}(z, L', B')$, compute the ex-ante default and borrowing policy functions of the planner using (45) and (46). Update the planner public borrowing and default policy functions.
- Compute the ex-ante value function $W^{SP}(z, L, B)$ using (47).
- Update the schedule of public bonds $\boldsymbol{Q}^{SP}(z, L', B')$ using (48).
- Repeat until convergence in $W^{SP}(z, L, B)$,$B^{SP\prime}(z, L, B, L')$, and $\boldsymbol{Q}^{SP}(z, L', B')$ is achieved.

F Particle filter method

This appendix details the particle filter method used to conduct the counterfactual exercises of section 5. It follows closely the approach presented in Bocola and Dovis (2019). As noted in the main text, the state space representation of the model is:

$$\boldsymbol{Y}_t = g(\boldsymbol{S}_t) + e_t \tag{49}$$

$$\boldsymbol{S}_t = f(\boldsymbol{S}_{t-1}, \varepsilon_t). \tag{50}$$

In this formulation, the first equation captures the measurement error e_t, a vector of i.i.d. normally distributed errors with mean zero and a diagonal variance-covariance matrix Σ. The vector of observable, $\boldsymbol{Y}_t$, includes average private and public debt as share of GDP, detrended tradable output, the share of nonperforming loans, and interest rate spreads on public bonds. The second equation describes the law of motion of the baseline model state variables $\boldsymbol{S}_t = [L_t, B_t, y^T_{t-1}, \pi_{t-1}, \kappa_{t-1}]$. The vector ε_t corresponds to the innovations in the AR 1 process of the three structural shocks $[y^T_t, \pi_t, \kappa_t]$.

$$
\begin{aligned}
y^T_t &= \exp(\rho^y \ln y^T_{t-1} + \varepsilon^y_t) \\
\pi^T_t &= \exp((1-\rho^\pi)\bar{\pi} + \rho^\pi \ln \pi_{t-1} + \varepsilon^\pi_t) \\
\kappa_t &= (1-\rho^\kappa)\bar{\kappa} + \rho^\kappa \kappa_t + \varepsilon^\kappa_t
\end{aligned}
$$

Since we did not observe any defaults in the time periods considered we use the repayment policy functions to compute the transitions. Using the notation of section 3 the evolution of private and public debt in the first exercise is then:

$$
\begin{aligned}
L_{t+1} &= \mathcal{L}'(s_t, L_t, B_t) = \mathcal{L}'(y^T_t, \pi_t, \kappa_t, 0, L_t, B_t) \\
B_{t+1} &= \mathcal{B}'(s_t, L_t, B_t) = \mathcal{B}'(y^T_t, \pi_t, \kappa_t, 0, L_t, B_t)
\end{aligned}
$$

In the first exercise all taste shocks are set to to zero. In the second exercise, we still focus on repayment but this time we select the taste shocks to match public debt exactly to it's data counter part and let private debt the respond endogenously:

$$
\begin{aligned}
L_{t+1} &== L^{data}_{t+1} \\
B_{t+1} &= \tilde{B}'(y^T_t, \pi_t, \kappa_t, L_t, B_t, 0, L^{data}_{t+1}, \tilde{T}(s_t, L_t, L^{data}_{t+1}))
\end{aligned}
$$

These transitions are summarized in function $f(\cdot)$ for each exercise. Similarly we can generate numerical solutions to compute the model counterparts to debt to output ratios and the public spreads and summarize them in $g(\cdot)$.

Let $\boldsymbol{Y}^t = [\boldsymbol{Y}_1, ..\boldsymbol{Y}_t]$, and denote by $p(\boldsymbol{S}_t|\boldsymbol{Y}^t)$ the conditional distribution of the state vector given a history of observations up to period t. In general there is no analytical solution for the density function $p(\boldsymbol{S}_t|\boldsymbol{Y}^t)$. The particle filter method approaches this density by using the fact that the conditional density of $\boldsymbol{Y}_t$ given $\boldsymbol{S}_t$ is Gaussian. It consists of finding a set of pairs of states and weights $\{S^i_t, \tilde{w}^i_t\}^N_{i=1}$ such that for all function $h(\cdot)$:

$$
\frac{1}{N}\sum_{i=1}^{N} h(\boldsymbol{S}^i_t)\tilde{w}^i_t \underset{a.s}{\longrightarrow} \mathbb{E}[h(\boldsymbol{S}_t)|\boldsymbol{Y}^t].
$$

This approximation can then be used to obtain the weighted average path of the state vector over the sample. The states selected S^i_t are called particles and $\tilde{w}^i_t$ corresponds to their weight. To construct this set we follow the algorithm proposed by Kitagawa (1996).

Step 1: Initialization Set $t = 1$ and $\forall i$ $\tilde{w}^i_0 = 1$, draw S^i_0 from the ergodic distribution of the baseline model.

Step 2: Transition For each $i = 1..N$ compute the state vector $\boldsymbol{S}^i_{t|t-1}$ given vector $\boldsymbol{S}^i_{t-1}$ by drawing innovations for the fundamental shocks from the calibrated distributions and using the policy functions summa zed in $f(\cdot)$.

Step 3: Filter Assign to each particle $\boldsymbol{S}^i_{t|t-1}$ the weight

$$w^i_t = p(\boldsymbol{Y}|\boldsymbol{S}^i_{t|t-1})\tilde{w}^i_{t-1}$$

where $p(\boldsymbol{Y}|\boldsymbol{S}^i_{t|t-1})$ is a multivariate Normal density.

Step 4: Rescale & Resample Rescale the weights $\{w^i_t\}$ so that they add up to one, and denote these new weights $\{\tilde{w}^i_t\}$. Sample with replacement N values of the state vector from the set $\{\boldsymbol{S}^i_{t|t-1}\}$ using $\{\tilde{w}^i_t\}$ as sample weights. Denote this draws $\{\boldsymbol{S}^i_t\}$. Set $\tilde{w}^i_t = 1\ \forall i$. If $t < T$ set $t = t + 1$ and go to Step 2. Otherwise stop.

In both exercises, it is assumed that measurement error associated with y^T_t and π_t is zero, as such the variance of the measurement error is set to zero for these variables in the measurement equation and the innovations ε^y_t and ε^π_t are set to match the empirical counterparts exactly. Since κ_t has no empirical counterpart, the algorithm help us find the most likely path using its effects on debt aggregates and the spreads. As in Bocola and Dovis (2019) the filter is tuned with $N = 100,000$.

Equipped with a set of particles and weights $\{S^i_t, \tilde{w}^i_t\}^N_{i=1}$ and the policy functions summarized in $g(\cdot)$ one can approximate the model predictions plotted in figures 11 and 12. As an example for all $t = [2008..2015]$ the predicted interest rate spread, $spr^{Baseline}_t$ at time t is:

$$spr^{Baseline}_t = \sum_i^N \tilde{w}^i_t[\frac{\delta - \delta\boldsymbol{Q}(S^i_t)}{\boldsymbol{Q}(S^i_t)} - r]$$

Similar weighted averages are computed for the debt to output ratio and the exogenous shocks. When computing objects for the social planner the function $g^{SP}(\cdot)$ is used instead.