A Macroprudential Theory of Foreign Reserve Accumulation *

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Abstract

We propose a macroprudential theory of foreign reserve accumulation that can rationalize the secular trends in public and private international capital flows. In middle-income countries, the increase in international reserves has been associated with elevated private capital inflows, both in the aggregate and in the cross-section, and economies with a more open capital account have accumulated more reserves. We present an open economy model of financial crises that is consistent with these features. We show that the optimal reserve management policy leans against the wind, raising gross private borrowing while improving the net foreign asset position and reducing the exposure to crises.

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JEL Classifications: E58, F31, F32, F34, F51

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1 Introduction

Central banks' holdings of international reserves have nearly quadrupled since the wave of financial globalization of the 1990s. Yet despite an extensive literature, accounting for this surge and the large variation in reserve holdings across countries has remained elusive. In this paper, we propose a simple theory of foreign reserve accumulation based on a macroprudential motive and show that it can quantitatively account for the buildup of international reserves while being consistent with salient cross-sectional patterns of capital flows.

Our theory is motivated by the intertwined relationship between foreign reserves and private external debt prevalent among middle-income countries. Four facts, which we document in Section 2, guide our analysis. First, at the aggregate level, the secular increase in foreign reserves has occurred at the same time as an increase in private external debt. Second, there is a positive association between the growth of reserves and debt, both over time and across countries. Third, reserves and private external debt accumulation appear to be procyclical for most countries. Fourth, reserve holdings are larger in economies with a more open capital account, a fact that resonates with the influential work by Ilzetzki, Reinhart and Rogoff (2019) linking the recent global accumulation of international reserves to the overall reduction in capital controls.

We argue that these facts point to a hypothesis linking international reserves to the government's prudential management of private capital flows. Few models of international capital flows, however, study explicitly the interaction between private and public capital flows. In fact, most of the literature has focused either on private or public flows, or considered a single borrowing agent without distinguishing between the two categories of flows. From an empirical point of view, distinguishing private and public capital flows has proved central to understanding key secular trends (Alfaro, Kalemli-Özcan and Volosovych, 2014). A first contribution of our paper is to provide a framework that can quantitatively speak to the evolution of private and public capital flows within a unified setup.

The environment we consider is a two-sector small open economy model with incomplete markets and inefficient private borrowing. The framework builds on a workhorse model of sudden stops and capital flows in emerging markets (e.g., Mendoza, 2002; Bianchi, 2011; Schmitt-Grohé and Uribe, 2017). A key feature of the model is that households' private borrowing is limited by an occasionally binding credit constraint that depends on income and links the borrowing capacity to the real exchange rate. In this setup, when an adverse shock hits and the economy is highly leveraged, households hit their credit constraint and become unable to smooth consumption. The contraction in spending leads to a depreciation of the real exchange rate, which further tightens the borrowing constraint and leads to a "sudden stop" in capital inflows. Our key departure from

the literature is to allow for the accumulation of a non-state contingent asset, which we refer to as international reserves, and to examine the implications for gross capital flows and optimal policy.

We show that while reserves provide a liquidity buffer to mitigate the contraction of consumption in a crisis, households do not internalize their general equilibrium benefits. Moreover, we demonstrate that the constrained-efficient allocation (i.e., the allocations that would prevail if the government were to make all financial decisions on behalf of private agents) can be implemented via a government reserve policy. When households deleverage, they fail to internalize how the contraction in their spending leads to a real exchange rate depreciation, further tightening economy-wide borrowing constraints. As a result, they do not face proper incentives to accumulate reserves in good times, when the credit constraint is not binding. A contribution of our paper is then to provide a theory of why it is the government rather than the private sector that must accumulate reserves.

Under Ricardian equivalence, reserve accumulation would be neutral as households would simply borrow more to offset the increase in reserve holdings by the government (Barro, 1974). When households' borrowing constraint is not binding, a small accumulation of international reserves by the government is indeed neutral. However, once the government accumulates a large enough amount of reserves, households become borrowing constrained and are unable to offset the government's reserve accumulation, thus breaking Ricardian equivalence. Hence, the very same credit constraint that makes households overborrow in good times, relative to the constrained-efficient allocation, also makes the reserve accumulation by the government effective. While gross debt increases under this government policy, the economy's net foreign asset position improves, leading to a reduction in the frequency and severity of sudden stops relative to the laissez-faire outcome.[1] A quantitative analysis of the model shows that the macroprudential motive for reserves can go a long way towards accounting for the intertwined relationship between private and public capital flows observed in the data. In particular, model simulations can account for the four aforementioned facts. The model generates (1) the observed upward trend in reserves and debt; (2) the positive association between yearly increases in reserves and debt in the cross-section; (3) the procyclicality of debt and reserves over the business cycle; and (4) the positive correlation between the degree of financial liberalization and reserves across countries.

We highlight that our theory is concerned with reserve accumulation by net debtor economies, where reserves serve a precautionary, macroprudential role. It is not intended to explain reserve accumulation by creditor countries such as China, nor the low-return puzzle associated with governments holding low-yielding reserves while issuing higher-yielding sovereign debt. In our

[1] See Loayza, Schmidt-Hebbel and Servén (2000) for evidence on the failure of Ricardian equivalence in the cross-section of countries.

framework, reserves and private borrowing carry the same interest rate, so the analysis abstracts from sovereign spread wedges between public liabilities and reserve assets.

Literature. Our paper adds to the extensive literature on capital controls and reserve accumulation by showing how an overborrowing externality provides a rationale for both policies. We demonstrate that, despite working through different channels, capital controls and reserve accumulation can ultimately result in equivalent welfare outcomes.

The idea of a precautionary motive for reserves has a long tradition in international macroeconomics (Kenen and Yudin, 1965; Heller, 1966, Clower and Lipsey, 1968; Clark, 1970; and Kelly, 1970). More recently, precautionary theories of reserves have focused on shocks to income or shocks to countries' access to credit markets, but in the context of models with a single decision-maker controlling all external financial decisions.[2] This literature has hence remained silent on the question of why it is the government that has to accumulate reserves. Our paper tackles this question and underscores an externality explaining why private agents may not have incentives to accumulate reserves on their own.

Few papers model jointly private and public capital flows in quantitative settings. A notable exception is Benigno, Fornaro and Wolf (2022), who consider a model in which reserves held by the government are motivated by the presence of a learning-by-doing externality in the tradable sector. They show that in the absence of industrial policies, accumulating reserves is desirable to undervalue the real exchange rate and foster export-led growth. Our work is complementary in that it articulates a role for reserve accumulation based on a macroprudential motive. Moreover, we examine optimal policy and show that the macroprudential motive can go a long way towards accounting for the observed levels of reserves and the interaction between private and public capital flows.

Our paper also relates to the literature that studies foreign exchange (FX) interventions in the presence of limits to international arbitrage. Examples include Cavallino (2019), who shows how FX interventions can deal with dynamic terms of trade externalities and capital account shocks, Amador, Bianchi, Bocola and Perri (2020), who show that reserve accumulation is needed to implement exchange rate policies at the zero lower bound, and Fanelli and Straub (2020), who characterize optimal policies when real exchange rate fluctuations lead to distributional consequences. While a common theme in these papers is that international intermediaries have limited leverage capacity, building on the work of Gabaix and Maggiori (2015), our focus is

[2]See, for example, Durdu, Mendoza and Terrones (2009), Caballero and Panageas (2008), Jeanne and Rancière (2011), Bianchi, Hatchondo and Martinez (2018), Bianchi and Sosa-Padilla (2024), Aizenman and Lee (2007), Bacchetta, Benhima and Kalantzis (2013), Hur and Kondo (2016), and Jeanne and Sandri (2017).

instead on frictions in domestic financial markets.[3] In addition, a key distinction of our paper is that we study the scope for reserve accumulation owing to financial stability, a motive notably raised by Calvo (2006) and Obstfeld, Shambaugh and Taylor (2010). In this respect, our paper is complementary to Bocola and Lorenzoni (2020), who show that reserves can enhance the credibility of lender of last resort policies.

Basu, Boz, Gopinath, Roch and Unsal (2020) also study capital controls and FX interventions within a unified framework. In their model, the rationale for capital controls is an overborrowing externality while the rationale for FX interventions is to reduce borrowing costs and improve risk-sharing.[4] In contrast, we show how an overborrowing externality generates a rationale for both FX interventions and capital controls.

Our paper also relates to the literature on financial crises and macroprudential policy. This literature has shown how capital controls can correct pecuniary externalities that generate excessive systemic risk (e.g., Lorenzoni, 2008; Bianchi, 2011; Dávila and Korinek, 2017; Bianchi and Mendoza, 2018; Jeanne and Korinek, 2018).[5] We contribute to this literature by showing how international reserves can serve as a macroprudential policy tool. Our results shed light on the observation by Ilzetzki, Reinhart and Rogoff (2019) of a rise in global reserves alongside increasing capital mobility.

Our work also connects with several studies that analyze the interaction between ex-ante and ex-post policies. The two most closely related ones are Benigno, Chen, Otrok, Rebucci and Young (2013) and Schmitt-Grohé and Uribe (2021).[6] In Benigno et al. (2013), the government has access to a richer set of tax instruments, enabling it to relax borrowing constraints ex-post, which results in more borrowing ex-ante compared to the laissez-faire economy. In Schmitt-Grohé and Uribe (2021), the optimal government intervention induces more borrowing relative to competitive equilibria driven by self-fulfilling pessimistic beliefs. In contrast with these studies, our model distinguishes between private and official flows. While we also find that under the optimal intervention, households borrow more, the accumulation of reserves ultimately improves the net foreign asset position.

Finally, our paper is related to a large empirical literature on capital flows. Particularly relevant is the empirical work on the precautionary motive for reserves (e.g., Edwards, 1983; Frankel and

[3] For a comprehensive review, see Maggiori (2021).

[4] See also Itskhoki and Mukhin (2022) for a framework where FX interventions can lower borrowing costs by reducing the exchange rate risk exposure of arbitrageurs.

[5] The scope for prudential policies can also emerge from aggregate demand externalities (see e.g., Schmitt-Grohé and Uribe, 2016; Farhi and Werning, 2016; Korinek and Simsek, 2016). See Bianchi and Lorenzoni (2021) for a review of the literature on prudential policies.

[6] See also Bianchi (2016), Bornstein and Lorenzoni (2018), Jeanne and Korinek (2020), and Arce, Bengui and Bianchi (2023).

Saravelos, 2012; Bussiere, Cheng, Chinn and Lisack, 2013; Calvo, Izquierdo and Loo-Kung, 2013). Our empirical and theoretical analysis emphasizes the interaction between private and public capital flows and the importance of considering gross positions, as stressed in Obstfeld (2012).

The paper is organized as follows. Section 2 outlines the motivating facts. Section 3 presents the model and the theoretical results. Section 4 contains a quantitative analysis, and Section 5 concludes.

2 Motivating facts: reserves, debt, and capital mobility

In this section, we present empirical evidence on international reserves and private external debt that illustrates the intertwined relationship between these two variables. We use data for middle-income countries from 1980 to 2015.[7] The data for private external debt are from the International Debt Statistics collected by the World Bank and measure private external debt as non-publicly guaranteed external debt.[8]

We summarize the evidence in four facts:

FACT 1: *Over the past three decades, there has been a substantial increase in private external debt and international reserves.* Figure 1 shows the evolution of the GDP-weighted average of private external debt and reserves from 1980 to 2015.[9] Until 1990, both international reserves and private external debt were below 5 percent of total GDP for the average middle-income country. By 2015, reserves and private external debt reached, respectively, 16 percent of GDP and 12 percent of GDP. It is worth noting that the sharp rise in private external debt contrasts with the decline in publicly guaranteed external debt (PGD) in the countries in our sample. Over the same period, PGD decreased from 27 percent of GDP in 1980 to 14 percent of GDP in 2015.

FACT 2: *There is a positive association between the growth of reserves and debt, both over time and across countries.* To assess the relationship between growth in reserves and growth in private

[7] The complete list of countries, based on data availability and other considerations detailed in Appendix A, is Argentina, Bolivia, Brazil, Cameroon, Colombia, Costa Rica, Ecuador, Egypt, El Salvador, Guatemala, Honduras, India, Indonesia, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Peru, Philippines, Sri Lanka, Thailand, Tunisia, Turkey, and Venezuela.

[8] An advantage of using data from the International Debt Statistics is that it allows us to differentiate publicly guaranteed debt (PGD) and non-PGD. This distinction is important, as some middle-income countries in our sample have large publicly owned companies that issue debt internationally.

[9] This trend also holds when we look at simple averages. Notice that Figure 1 excludes China.

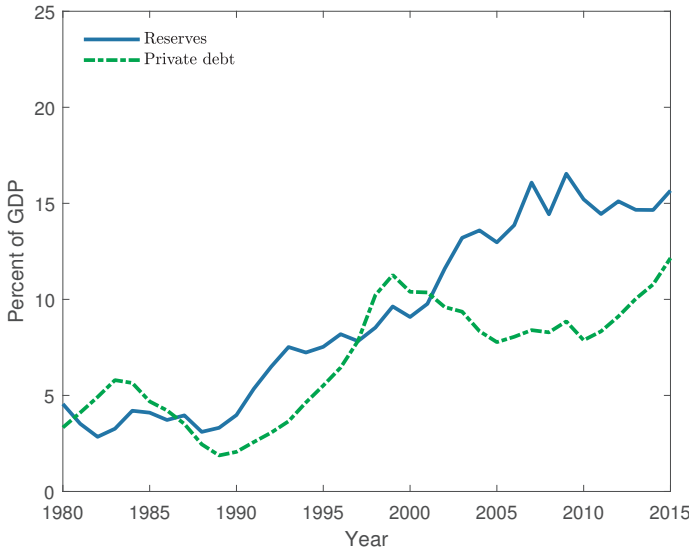


Figure 1: Evolution of reserves and private debt (GDP-weighted average)

external debt at a more granular level, we estimate variants of the following panel regression:

$$\Delta a_{it} = \beta \Delta b_{it} + \gamma x_{it} + \varepsilon_{it}, \tag{1}$$

where Δa_{it} and Δb_{it} denote the yearly changes in the foreign reserves-to-GDP ratio and private external debt-to-GDP ratio for country i, and x_{it} denotes control variables. Our coefficient of interest is β. We consider the results with pooled ordinary least squares (OLS) regressions as well as country and time fixed effects.

Table 1 reports the estimation results, along with standard errors clustered at the country level.[10] In all cases, the coefficient on private external debt is positive and statistically significant at the 1 percent significance level, indicating a robust statistical association between changes in private external debt and changes in reserves. The estimation with a country fixed effect only (column 2) confirms that changes in private external debt and changes in reserves are significantly positively correlated within country units. Similarly, the relationship is preserved when we allow for time fixed effects only (column 3), confirming that the positive association also holds in the cross-section. In other words, in a given year, countries that accumulate more private external

[10] In all the regressions we include a constant and control for the current account-to-GDP ratio. The estimates of the parameter of interest are also positive and statistically significant when we control for the net changes in the PGD-to-GDP ratio and real GDP growth.

debt also tend to accumulate more reserves.[11,12]

Table 1: Changes in Reserves-to-GDP Ratios on changes in Private External Debt-to-GDP Ratios

	(1) Reserves	(2) Reserves	(3) Reserves	(4) Reserves
Private External Debt	0.261***	0.276***	0.254***	0.266***
	(0.0421)	(0.0420)	(0.0470)	(0.0460)
Current Account	0.175***	0.251***	0.162***	0.251***
	(0.0237)	(0.0294)	(0.0265)	(0.0356)
Observations	875	875	875	875
Countries	25	25	25	25
Fixed Effects	No	Country	Time	Country+Time

Note: Standard errors clustered at the country level in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

FACT 3: *The accumulation of reserves and private external debt is procyclical for most countries.* Appendix A.3 provides more details on this fact. Figure A.1 displays correlations between, on the one hand, a country's real GDP growth and, on the other hand, the growth rates of its reserves (left panel) or of its private external debt (right panel). Both of these correlations are positive for a majority of countries. As shown in Table A.5, the positive association between output growth and reserves growth, and output growth and private debt growth is also apparent when using a pooled OLS estimation.

FACT 4: *Reserve holdings are larger in economies with a more open capital account.* Figure 2 shows a scatter plot of the Chinn and Ito (2008) index of capital account openness and the average ratio of reserves-to-GDP. It shows a positive correlation between reserves and capital account openness. In other words, emerging countries that impose significant controls on international private flows of capital tend to have relatively smaller ratios of reserves-to-GDP than countries with more liberalized capital accounts. In Appendix A.3, we verify that a positive correlation between reserves and capital account openness is present in a pooled OLS regression (see also Aizenman and Lee, 2007 and Bussiere et al., 2013 for related evidence).

To summarize the empirical evidence that motivates our theoretical analysis, the data show

[11] These findings complement Broner, Didier, Erce and Schmukler (2013)'s results on the positive correlation between inflows of private debt and reserves over time, as well as Obstfeld, Shambaugh and Taylor (2010)'s results on the positive association between private *domestic* debt and reserves.

[12] The positive association between reserves and private external debt is also obtained when we express all variables in US dollars as opposed to percents of GDP and when we estimate the model using panel feasible generalized least squares (FGLS) estimation, as shown in Appendix A.3.

that reserves and private external debt are deeply intertwined. In particular, we highlight four facts: (i) a substantial increase in private external debt and international reserves in the aggregate; (ii) a positive correlation between reserves and private external debt in the cross-section; (iii) the procyclicality of reserve and private external debt accumulation; (iv) reserve holdings are larger in economies with a more open capital account.

These observations indicate positive associations, but they do not point to causality in *either* direction. We next propose a theory that sheds light on the interplay between private external debt and reserves and delivers dynamics consistent with the four aforementioned facts.

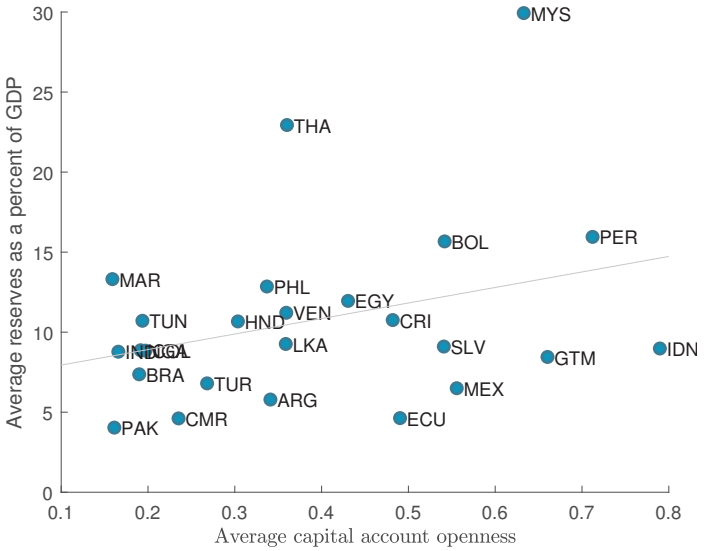


Figure 2: Average 1980–2015 reserves and Chinn and Ito (2008) capital account openness

3 Model

We consider a dynamic small open-economy model with tradable and non-tradable goods. The economy is populated by a continuum of identical households that borrow externally subject to an occasionally binding borrowing constraint. We describe first the households' problem, and then we analyze the competitive equilibrium and the role of international reserves.

3.1 Households' problem

Households' preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{2}$$

where $\mathbb{E}_0$ is the expectation operator conditional on date 0 information; $0 < \beta < 1$ is a discount factor; $u(\cdot)$ is a standard increasing, concave, and twice continuously differentiable function satisfying the Inada conditions; and consumption c_t is an Armington-type constant elasticity of substitution (CES) aggregator with elasticity of substitution $1/(\eta + 1)$ between tradable goods c_t^T and non-tradable goods c_t^N, given by

$$c_t = \left[\omega \left(c_t^T\right)^{-\eta} + (1-\omega)\left(c_t^N\right)^{-\eta}\right]^{-\frac{1}{\eta}}, \qquad \text{with} \quad \eta > -1, \omega \in (0,1).$$

In each period, households receive a random endowment of tradable goods y_t^T and a fixed endowment of non-tradable goods y^N. We use the tradable good as the numeraire.

Households can borrow (or save) using a one-period non-state-contingent bond b_{t+1} denominated in units of tradables paying an interest rate R_t, which is exogenously determined in international capital markets and may vary stochastically.[13] Their budget constraint, in units of tradables, is given by

$$c_t^T + p_t^N c_t^N - \frac{b_{t+1}}{R_t} = y_t^T + p_t^N y^N - b_t - T_t, \tag{3}$$

where p_t^N is the price of non-tradable goods and T_t is a lump-sum tax. In addition, households face a credit constraint given by

$$\frac{b_{t+1}}{R_t} \leq \kappa_t \left(y_t^T + p_t^N y^N\right). \tag{4}$$

This credit constraint captures in a parsimonious way the empirical fact that current income is critical in determining credit-market access (see e.g. Jappelli, 1990; Lian and Ma, 2021), and it has been shown to be important for accounting for the dynamics of capital flows in emerging markets (e.g., Mendoza, 2002). Non-tradable goods enter the collateral constraint because while foreign creditors do not value these goods directly, they can seize them in the event of default and

[13] Assuming no foreign inflation, it is equivalent to denominating the bonds in foreign currency, capturing the liability dollarization phenomenon.

sell them in exchange for tradable goods on the domestic market.[14] We allow for shocks to the parameter κ_t, which we refer to as a financial shock. One interpretation of this shock is that it captures fluctuations in lenders' perceptions about households' ability to repay or in the country's institutional contract enforcement.

Households choose consumption and borrowing to maximize their utility (2) subject to their budget (3) and credit constraint (4), taking prices and taxes as given. Their optimality conditions are given by

$$p_t^N = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{\eta+1}, \tag{5}$$

$$\lambda_t = u_T(t), \tag{6}$$

$$\lambda_t = \beta R_t \mathbb{E}_t \lambda_{t+1} + \mu_t, \quad \text{with } \mu_t = 0 \text{ if } \frac{b_{t+1}}{R_t} < \kappa_t \left(y_t^T + p_t^N y^N\right), \tag{7}$$

where $u_T(t)$ is shorthand notation for $\frac{\partial u}{\partial c}\frac{\partial c}{\partial c^T}$ and μ_t denotes the non-negative Lagrange multiplier on the borrowing constraint. Condition (5) is a static optimality condition equating the marginal rate of substitution between tradable and non-tradable goods to their relative price. Condition (6) equates the marginal utility of tradable consumption to the shadow value of current wealth, and Condition (7) is the household's Euler equation for debt. When $\mu_t > 0$, the marginal utility benefits from increasing tradable consumption today exceed the expected marginal utility costs from borrowing one unit and repaying next period.

3.2 Government

The government accumulates international reserves $A_{t+1} \geq 0$ that pay an interest rate R_t and finances them with lump-sum taxes and existing holdings of reserves A_t:

$$\frac{A_{t+1}}{R_t} = T_t + A_t. \tag{8}$$

[14] The credit constraint can be derived endogenously from a problem of limited enforcement under two assumptions. First, households can default at the end of the current period. Second, upon default, foreign creditors can seize a fraction κ_t of the current income, and households immediately regain access to credit markets. The current, rather than the future, price appears in the constraint because the opportunity to default occurs at the end of the current period, before the realization of future shocks (see Bianchi and Mendoza, 2018, for a derivation of a similar constraint).

3.3 Competitive equilibrium

The market clearing condition for non-tradable goods is

$$c_t^N = y^N. \tag{9}$$

We can now define a competitive equilibrium for any government policies. Given initial conditions A_0, b_0, and government policies $\{T_t, A_{t+1}\}_{t\geq 0}$, a *competitive equilibrium* is defined as a stochastic sequence of prices $\{p_t^N\}_{t\geq 0}$ and households' policies $\{c_t^T, c_t^N, b_{t+1}\}_{t\geq 0}$ such that (i) households maximize their utility (2) subject to the sequence of budget constraints (3) and credit constraints (4), taking as given prices and government policies; (ii) the government budget constraint (8) is satisfied; and (iii) the market for non-tradable goods clears (9).

Combining the household's budget constraint (3) with the government's budget constraint (8) and the non-tradable goods market clearing (9), we obtain the economy's consolidated resource constraint for tradable goods:

$$c_t^T + \frac{A_{t+1} - b_{t+1}}{R_t} = y_t^T + A_t - b_t. \tag{10}$$

This constraint illustrates that from the perspective of the resource constraint in the small open economy, official reserves and household-issued bonds are perfect substitutes. Absent the credit constraint (4), Ricardian equivalence would hold and the amount of foreign reserves accumulated by the government would be completely irrelevant. However, as we argue below, the presence of the credit constraint (4) makes reserve accumulation both *relevant* and *desirable*.

Remark on households' accumulation of reserves. Notice that we have not explicitly considered the accumulation of the reserve asset by households, but this is without loss of generality. When the borrowing constraint is binding, the return on reserves R_t is lower than the shadow return on borrowing, and so households' optimal choice of reserves is zero. When the borrowing constraint is not binding, households are indifferent between reserves and debt because both debt and reserves have the same maturity and deliver a risk-less return R_t in units of tradables.

3.4 Constrained efficiency

The competitive equilibrium is constrained inefficient due to a pecuniary externality. Households do not internalize that by borrowing more in the present and consuming less in the future, they

put downward pressure on the future price of non-tradables and thereby contribute to tightening other agents' future credit constraints. Following Bianchi (2011), we consider the problem of a constrained social planner who directly chooses the economy's debt subject to the borrowing constraint and allows goods markets to clear competitively. Assuming that shocks follow a first-order Markov process, the problem can be written recursively as follows:

$$V(b, y^T, R, \kappa) = \max_{b', c^T} u(c(c^T, y^N)) + \beta \mathbb{E} V(b', y^{T'}, R', \kappa') \tag{11}$$

subject to

$$b + c^T = y^T + \frac{b'}{R}, \tag{12}$$

$$\frac{b'}{R} \leq \kappa \left[y^T + \frac{1-\omega}{\omega} \left(\frac{c^T}{y^N} \right)^{\eta+1} y^N \right]. \tag{13}$$

where the substitution of the price of non-tradables expression (5) into (13) reflects the implementability constraints of the planner.

Using sequential notation, for convenience, the planner's Euler equation for debt is given by

$$\lambda_t = \beta R_t \mathbb{E}_t \lambda_{t+1} + \mu_t, \tag{14}$$

where λ_t and μ_t denote the Lagrange multipliers on (12) and (13). While equation (14) resembles the private Euler equation (7), a critical difference is that the shadow value of current wealth differs and is given by

$$\lambda_t = u_T(t) + \mu_t \Psi_t, \tag{15}$$

where Ψ_t denotes the equilibrium change in the collateral value associated with a marginal change in tradable consumption, defined as:

$$\Psi_t \equiv \kappa_t (p_t^N c_t^N)/(c_t^T)\,(1+\eta)\,. \tag{16}$$

The change in the value of collateral associated with a marginal change in tradable consumption is the product of three terms: the collateral parameter κ_t, the ratio of non-tradable to tradable expenditure, and the inverse of the elasticity of substitution.

The wedge between the planner's and the private marginal value of wealth captures how the planner internalizes that higher demand for consumption relaxes the economy's borrowing constraint. This wedge translates into an "overborrowing" externality whenever the credit constraint does not currently bind but is expected to bind with strictly positive probability in the next period.

3.5 Reserve accumulation

In this section, we demonstrate that the constrained-efficient allocations can be implemented using a policy for reserve accumulation. One potential advantage of the implementation with reserves relative to capital controls is the observation that leakages often undermine the effectiveness of the latter (Bengui and Bianchi, 2022). This may make reserve accumulation a more attractive policy to pursue in practice and can, in fact, rationalize why governments seldom resort to the use of capital controls (Fernandez, Rebucci and Uribe, 2015) and instead use reserves as a primary policy tool.

To establish our result, it is convenient to impose the following assumption.

Assumption 1. *Consumption is a Cobb-Douglas aggregator* $c = (c^T)^{\omega}(c^N)^{1-\omega}$*, and the credit constraint parameter satisfies* $\kappa_t(1-\omega)/\omega < 1$.

This assumption implies that $\Psi_t < 1$, for Ψ_t defined in (16), and guarantees that in any equilibrium, an increase in aggregate consumption by one unit does not relax the credit constraint by more than one unit. We return to the role played by this assumption later in this section, and emphasize that it is by no means necessary for our result to hold.

Our main normative result is summarized in the following proposition.

Proposition 1. *Suppose Assumption 1 holds. Consider the solution to the constrained-efficient planning problem* $\{c_t^{T\star}, b_{t+1}^{\star}, p_t^{N\star}\}_{t=0}^{\infty}$*. Then, given initial conditions* (b_0, A_0) *such that* $b_0^{\star} = b_0 - A_0$*, the competitive equilibrium features a tradable consumption allocation* $\{c_t^{T\star}\}_{t=0}^{\infty}$ *if the government follows the reserve policy* $\{A_{t+1}\}$ *given by*

$$\frac{A_{t+1}}{R_t} = \kappa_t \left(y_t^T + p_t^{N\star} y^N \right) - \frac{b_{t+1}^{\star}}{R_t} \quad \text{for all } t \geq 0. \tag{RP}$$

Proof. The proof is by construction. We will show that, given the sequence of prices $\{p_t^{N\star}\}_{t=0}^{\infty}$ and initial conditions, the sequence of consumption allocations $\{c_t^{T\star}, y^N\}_{t=0}^{\infty}$ satisfies the households' first-order conditions, which are both necessary and sufficient for optimality.

We start by guessing that given (RP), the households' credit constraint (4) holds with equality at all times:

$$\frac{b_{t+1}}{R_t} = \kappa_t \left(y_t^T + p_t^{N\star} y^N \right). \tag{17}$$

Combining (17) with (RP), we obtain

$$b_{t+1}^{\star} = b_{t+1} - A_{t+1}. \tag{18}$$

Substituting (18) into the tradable resource constraint (10) yields

$$c_t^T = y_t^T - (b_t - A_t) + \frac{b_{t+1}^\star}{R_t}. \tag{19}$$

Meanwhile, since $\{c_t^{T\star}, b_{t+1}^\star\}$ solve the constrained planning problem, we have

$$c_t^{T\star} = y_t^T - b_t^\star + \frac{b_{t+1}^\star}{R_t}. \tag{20}$$

Given the initial condition $b_0^\star = b_0 - A_0$, a comparison of (19) and (20) reveals that $c_t^T = c_t^{T\star}\ \forall t \geq 0$. That is, when households' borrowing policy satisfies (17) and reserves are set according to (RP), the constrained-efficient sequence of tradable consumption is consistent with the consolidated budget constraints of the household and the government. Notice that the non-negativity of A_{t+1} follows immediately from the reserve policy (RP) and the planner's credit constraint (13).

We are left to show that $c_t^T = c_t^{T\star},\ c_t^N = y^N$ satisfy the optimality conditions of the households. From conditions (14)-(15) characterizing the constrained-efficient allocation, we have

$$\mu_t^\star = u_T(t) - \beta R_t \mathbb{E}_t u_T(t+1) - \beta R_t \mathbb{E}_t \Psi_{t+1}^\star \mu_{t+1}^\star + \mu_t^\star \Psi_t^\star. \tag{21}$$

Rearranging the households' intertemporal Euler equation (7), we have that

$$\mu_t = u_T(t) - \beta R_t \mathbb{E}_t u_T(t+1). \tag{22}$$

Combining (21) and (22), we obtain

$$\mu_t = \beta R_t \mathbb{E}_t \Psi_{t+1}^\star \mu_{t+1}^\star + \mu_t^\star (1 - \Psi_t^\star) \geq 0, \tag{23}$$

where the non-negativity of μ_t follows from $\Psi_t^\star = \kappa_t(1-\omega)/\omega < 1$, given Assumption 1, and the non-negativity of $\mu_t^\star$. Together, the conjecture (17) and the fact that $\mu_t \geq 0$ ensure that the households' intertemporal Euler equation and complementary slackness condition are satisfied. That is, condition (7) holds. Finally, notice that the households' intratemporal condition (5) follows directly from the definition of the constrained-efficient allocation, implying that $c_t^N = y^N$ is also optimal. ∎

The proposition establishes that under the reserve accumulation policy (RP), the competitive equilibrium achieves the same level of consumption as in the constrained-efficient allocation—and therefore delivers the same welfare. When the government accumulates reserves, households take on more debt to maintain the same level of consumption, until the credit constraint becomes

binding. At that point, further increases in reserves generate a reduction in consumption and an increase in the net foreign asset position. The government then fine-tunes the amount of reserves to deliver the constrained-efficient net foreign asset position.

The reserve policy effectively puts private agents against their credit constraint whenever consumption in the laissez-faire economy would be above its level in the constrained-efficient allocation. The borrowing constraint only binds, however, when there is a strictly positive probability of a binding credit constraint in the subsequent period under the constrained-efficient allocation. In states in which the credit constraint is not expected to bind next period, the Lagrange multiplier associated with the constraint is zero. In fact, the constrained-efficient allocation can also be achieved by any alternative reserve policy satisfying $A_{t+1} \leq R_t \kappa_t \left(y_t^T + p_t^{N\star} y^N\right) - b_{t+1}^\star$ and such that the borrowing constraint does not hold with equality.[15] Intuitively, when the credit constraint does not bind, the anticipation that the constrained-efficient consumption allocation will prevail in the future leads households to pick the constrained-efficient consumption even without a government intervention.

In Appendix C, we also provide a dual result: the reserve policy that maximizes welfare in the competitive equilibrium yields the constrained-efficient allocation. In light of this result, in the remainder of the paper we occasionally refer to the implementation of the constrained-efficient allocation via reserve policy as the "optimal reserve policy" outcome.

Reserve depletion and liquidity value. As the expression (RP) indicates, when the credit constraint holds with equality in the constrained-efficient allocation, the government depletes its stock of reserves, setting $A_{t+1} = 0$. This result illustrates the liquidity value of reserves for the economy. The government accumulates reserves in good times to be used during crisis times. By rebating reserves to households during a crisis, it stabilizes consumption, raises the price of non-tradables, and reduces the amount of deleveraging. Because households do not internalize how a reserve buffer would generate positive general equilibrium effects during crises, it is the government that must accumulate the reserves.

To illustrate the importance of depleting reserves during a crisis, consider an alternative policy by which the government keeps a fraction ϕ of reserves: $A_{t+1} = \phi A_t$. Substituting this reserve policy into the economy's resource constraint (10) when the credit constraint (13) is binding yields

[15] This can be seen by noting that if $\mu_{t+1}^\star = 0$ in all future states and $\mu_t^\star = 0$, (23) implies that $\mu_t = 0$, implying that the credit constraint is slack in the competitive equilibrium. In our quantitative analysis, the indeterminacy of the reserve policy arises only in 2.4% of the simulations.

a level of tradable consumption of

$$c_t^T = \frac{(1+\kappa_t)\, y_t^T + \left[1 - \frac{\phi}{R_t}\right] A_t - b_t}{1 - \kappa_t \frac{1-\omega}{\omega}}.$$

Hence, maximizing current consumption—the planner's effective objective when the credit constraint binds—requires a full reserve depletion (i.e., setting $\phi = 0$). The above expression also clarifies why private households undervalue reserves in a crisis. While from an individual perspective, a unit of reserves provides enough resources to consume one additional unit of tradable goods, in equilibrium a unit of reserves raises tradable consumption by $1/[1 - \kappa_t(1-\omega)/\omega] > 1$.

The sharp reduction in reserves when the planner's credit constraint binds is consistent with the evidence that central banks use a large portion of reserves during crises (see, e.g., Broner et al., 2013). In many cases, however, reserves are not entirely depleted (Aizenman and Sun, 2012). A potential explanation for why central banks may choose to keep a positive level of reserves during crises is that policymakers may fear that losing large amounts of international reserves would send a bad signal to market participants. Alternative explanations are pursued by Bocola and Lorenzoni (2020) where reserves provide resources to buy capital and avoid a self-fulfilling collapse in banks' net worth and Barbosa-Alves, Bianchi and Sosa-Padilla (2024) where reserves provide liquidity to avoid a rollover crisis. Whereas the sole availability of reserves can implement the good equilibrium in those studies, here it is essential that the government actively uses the reserves during crises.

3.6 Discussion

3.6.1 Role of Assumption 1

Assumption 1 is sufficient for our reserve implementation to work, but it is by no means necessary. In Appendix B, we prove a more general version of Proposition 1, in which we relax Assumption 1—departing in particular from Cobb-Douglas preferences—and show that the reserve accumulation policy (RP) still implements the constrained-efficient allocation under an alternative weaker condition. As we show in the appendix, a necessary condition is

$$\mu_t^\star(\Psi_t^\star - 1) \leq \beta R_t \mathbb{E}_t \mu_{t+1}^\star \Psi_{t+1}^\star \tag{24}$$

where stars refer to variables evaluated at the constrained-efficient allocations.

The implications of a violation of condition (24) can be more easily understood by assuming

that $\mu^{\star}_{t+1} = 0$ in all successor states. In this case, when $\Psi^{\star}_t > 1$ and $\mu^{\star}_t > 0$, the planner is borrowing constrained but chooses a level of consumption that is higher than the unrestricted one (i.e., the level that would prevail at date t absent the date t credit constraint). This occurs because a low elasticity of substitution or a high κ_t (leading to a value of $\Psi_t > 1$) generates a non-convexity in the planner's problem such that the planner may be forced to choose between very low levels of consumption or very high ones. As can be seen from (23), such an allocation cannot be implemented with reserves because it would imply a negative Lagrange multiplier on the borrowing constraint for the household.[16] Intuitively, households would never choose an allocation such that $u_T(t) < \beta R_t \mathbb{E}_t u_T(t+1)$. Achieving this allocation would require a subsidy on borrowing in these states, and reserve accumulation alone would not be enough. Even though our implementation result would not hold in this case, reserves would remain an effective tool to reduce overborrowing ex ante.

Assumption 1 is related to the condition for self-fulfilling financial crises identified in Schmitt-Grohé and Uribe (2021). As they explain, equilibrium multiplicity may occur in this model because even though, for an individual agent, an increase in debt tightens the borrowing constraint, in equilibrium, an increase in aggregate debt may actually raise the borrowing capacity more than one-for-one and relax the borrowing constraint. Assumption 1 is sufficient to ensure that when aggregate borrowing increases by one unit, the borrowing capacity does not increase by more than one unit. As Schmitt-Grohé and Uribe show, this rules out multiple equilibria by which sharp drops in consumption can become self-fulfilling.

It is worth highlighting that our implementation result would still hold under multiplicity of equilibria, as long as (24) holds. However, the reserve policy would not be able to *uniquely* implement the constrained-efficient allocation. To understand why, consider a situation in which, for a given level of debt, reserves, and shocks, the economy features multiple equilibrium levels of current consumption. A planner that can directly choose the level of borrowing would choose the high consumption equilibrium, but it may not be able to implement it using reserve accumulation. If agents were to coordinate on the bad equilibrium, the government would deplete its reserves to raise consumption and support the real exchange rate, but this would not be sufficient to increase the borrowing capacity to reach the good equilibrium unless all households were to further increase their consumption simultaneously. Nevertheless, the macroprudential role of reserves remains intact. In fact, the possibility of being trapped in a bad equilibrium could give

[16] If the credit constraint is expected to bind in the following period (i.e., $\mu^{\star}_{t+1} > 0$ in some state at date $t+1$), the necessary condition is weaker. Intuitively, even though $\Psi^{\star}_t > 1$ indicates that a collective increase in borrowing is feasible, the planner may still choose a level of borrowing below the unconstrained level because it internalizes that more borrowing would tighten the constraint in the next period. As a result, given (23), we still have a positive Lagrange multiplier for households evaluated at the constrained-efficient allocation.

rise to an even more significant role for reserves ex ante.[17]

3.7 Practical implementation

In this section, we discuss several practical aspects regarding the use of reserve accumulation with a macroprudential approach.

3.7.1 Separation between fiscal and monetary authorities

We have assumed that there is a consolidated government that finances reserve accumulation with lump-sum taxes. In practice, however, reserve transactions are conducted by central banks that have no access to taxation power and instead finance reserve purchases by either issuing monetary or non-monetary liabilities or by selling other assets in their portfolios. We show next how these considerations can easily be incorporated into our model without changing our main insight.

We assume that the central bank and the fiscal authority face separate budget constraints. The central bank holds government bonds B^g and foreign reserves A and provides transfers T^{CB} to the fiscal authority. The fiscal authority collects lump-sum taxes and issues debt $\bar{B}^g$. Denoting by R_t^g the interest rate on government bonds, the budget constraints of the two authorities are respectively given by:

$$\frac{A_{t+1}}{R_t} + \frac{B_{t+1}^g}{R_t^g} + T_t^{CB} = A_t + B_t^g, \tag{25}$$

$$\bar{B}^g = \frac{\bar{B}^g}{R_t^g} + T_t + T_t^{CB}. \tag{26}$$

The household's budget constraint remains as before, except that now households can also accumulate government bonds.[18] We denote by b^g their holding of government bonds, which we assume to be non-negative. Equilibrium on the government bond market requires $B_t^g + b_t^g = \bar{B}^g$ for all t.

Proposition D3 in Appendix D.1 shows formally that the implementation of the constrained-efficient allocation with reserve accumulation carries over to this environment. Specifically, we focus on a configuration where the central bank can remit operating profits but cannot receive transfers from the fiscal authority, $T_t^{CB} \geq 0$, and we assume that it has a sufficiently large initial

[17] The parameterization we use for our quantitative analysis violates Assumption 1 but delivers a unique equilibrium, as in Bianchi (2011). Appendix E provides details on how we check numerically for the presence of multiplicity.

[18] That is, their budget constraint is $c_t^T + p_t^N c_t^N - \frac{b_{t+1}}{R_t} + \frac{b_{t+1}^g}{R_t^g} = y_t^T + p_t^N y^N - b_t + b_t^g - T_t$.

level of net worth. We show that under these conditions, the fiscal authority finances the servicing of its debt with central bank transfers and lump-sum taxes from households, while the central bank finances its purchase of foreign reserves by selling government bonds and rebates any profits to the fiscal authority.[19]

3.7.2 Distortionary taxes

In our baseline model, the implementation of the macroprudential intervention requires that the government has access to taxes to finance the acquisition of reserves.[20] Crucially, we have assumed that taxes are lump sum. In Appendix D.2, however, we relax this assumption and introduce a distortionary cost of raising taxes to finance the reserve accumulation. We show that distortionary costs introduce two wedges in the government's Euler equation: one capturing the current-period losses from taxation and another reflecting the expected future costs of raising taxes. While distortionary taxes prevent the decentralization of the constrained-efficient allocation, we find that a sizable accumulation of reserves remains desirable for standard distortionary costs calibrations.[21]

3.7.3 Implementation via a feedback rule

In this section, we show that the constrained-efficient allocation can be implemented using a simple *feedback rule* that directly specifies reserve policy as a function of the private sector's borrowing choice. Feedback rules are common in the study of monetary policy (e.g., the Taylor rule) and under some circumstances can achieve the same outcomes as the state-contingent optimal policy (see e.g. Woodford, 2007). It turns out that a similar equivalence applies to our model.

Proposition 2. *Suppose Assumption 1 holds and initial conditions* (b_0, A_0) *are such that* $b_0^\star = b_0 - A_0$. *Then, the constrained-efficient allocation is achieved if the reserves are set according to the feedback rule given by*

$$\mathcal{A}_{t+1}(b_{t+1}) = b_{t+1} - b_{t+1}^\star. \tag{27}$$

Proof. See Appendix B.2. ∎

[19] Note that we are not allowing foreign investors to accumulate government bonds. If investors had access to the domestic bond market, they would earn a rent at the expense of the small open economy, generating an extra cost from the intervention for the small open economy (see Amador et al., 2020 and Fanelli and Straub, 2020).

[20] This also applies to the extension considered above, where the fiscal and monetary authorities are separated. This is because the consolidated government needs to finance the difference between the interest rate on reserves and government bonds (often referred to as quasi-fiscal costs, see Calvo, 1991).

[21] Another crucial element for the decentralization result is that the interest rate on reserves equals the interest rate on bonds. See Appendix D.4 for an extension where the interest rate on reserves is lower than the one on bonds.

The feedback rule provides a simple, yet clear policy insight: the government should save an amount equal to the gap between the private sector's borrowing b_{t+1} and the constrained-efficient level of borrowing $b^\star_{t+1}$.

To understand the mechanics of the feedback rule and the strategic interactions between the government and households, consider a scenario in which the constrained-efficient allocation is implemented from date $t+1$ onward and let us focus on the outcome at date t. Denote by $\tilde{c}^T_t$ the level of tradable consumption the household would choose at date t in the absence of any reserve intervention, and by $\tilde{b}_{t+1}$ the associated level of private borrowing. Meanwhile, denote by $c^{T\star}_t$ and $b^\star_{t+1}$ the constrained-efficient tradable consumption and net borrowing. Given the overborrowing result, we have $\tilde{c}^T_t \geq c^{T\star}_t$ and $\tilde{b}_{t+1} \geq b^\star_{t+1}$.

Let us now examine how households respond to the purchases of reserves A_{t+1} at date t. To stay on their Euler equation (7), households adjust their borrowing according to

$$\mathcal{B}_{t+1}(A_{t+1}) = \begin{cases} A_{t+1} + \tilde{b}_{t+1} & \text{for } A_{t+1} < \bar{A}_{t+1} \\ R_t \kappa_t \dfrac{\frac{1}{\omega} y^T_t - \frac{1-\omega}{\omega}\left(b_t + \frac{A_{t+1}}{R_t}\right)}{1 - \kappa_t \frac{1-\omega}{\omega}} & \text{for } A_{t+1} \geq \bar{A}_{t+1}, \end{cases}$$

where $\bar{A}_{t+1} \equiv R_t \kappa_t \left(y^T_t + \frac{1-\omega}{\omega}\tilde{c}^T_t\right) - \tilde{b}_{t+1}$ denotes the threshold of reserve purchases at which point the households' borrowing constraint becomes binding. For $A_{t+1} < \bar{A}_{t+1}$, households react to the lump-sum tax (expected to be offset by a positive future transfer) by a one-to-one increase in debt, following a Ricardian equivalence type of logic. For $A_{t+1} > \bar{A}_{t+1}$, the private debt level required to offset the tax is so large that it violates the household's credit constraint. In fact, above the threshold, more reserves contract the borrowing capacity of the economy and lead to *less* private debt rather than more private debt.

What is the level of reserves that the government needs to accumulate to implement a consumption of $c^{T\star}_t$ and a net borrowing of $b^\star_{t+1}$? Figure 3 illustrates how the interaction between the government's policy and the households' response determines the necessary level of reserves. The solid blue line represents the private sector's best response $\mathcal{B}_{t+1}(A_{t+1})$, and the dashed red line represents the government's policy $\mathcal{A}_{t+1}(b_{t+1})$ described by (27). Notice that the fact that households borrow more in response to the accumulation of reserves leads the government to accumulate even more reserves. Equilibrium is reached when both the private sector and the government play their best responses, i.e., when $\mathcal{A}_{t+1}(b_{t+1})$ and $\mathcal{B}_{t+1}(A_{t+1})$ intersect. At that point, official reserves are positive and private indebtedness has increased, but the economy's net foreign asset position has improved relative to the level that would have resulted from households'

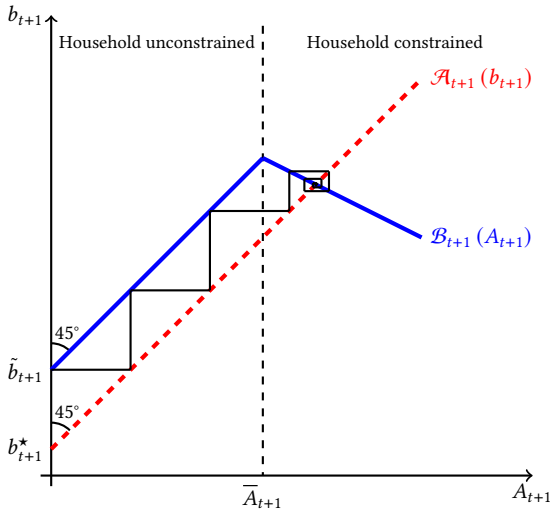


Figure 3: Illustration of the implementation of constrained-efficient allocation via feedback rule for reserves when the borrowing constraint is slack for the planner and $c_t^{T\star} < \tilde{c}_t^T$.

borrowing choice absent any reserve intervention at date t.[22]

3.8 Policy Discussion

The credit constraint is crucial for our normative analysis. First, it induces households to borrow less and hinders their ability to smooth consumption in response to adverse shocks. In addition, the dependence of the credit constraint on the real exchange rate implies that households borrow too much from a constrained-efficiency perspective (Bianchi, 2011). Households do not internalize that by borrowing more, they depreciate the real exchange rate in the future and thereby make the borrowing constraint tighter for other agents in the economy. Accordingly, a tax on debt can reduce borrowing and implement the constrained efficient outcome, as previously shown in the literature.

Our paper shows that a policy of reserve accumulation can implement the same outcome. In other words, the same friction that leads to capital controls being optimal creates a role for foreign exchange interventions. At the heart of our result is that once the reserve accumulation is sufficiently large, Ricardian equivalence ceases to hold. That is, starting from a state in which households' borrowing constraint is slack, accumulating reserves leads to a one-to-one increase in private debt. However, once the intervention becomes large enough, households can no longer increase borrowing to smooth their consumption. The government policy precisely exploits this

[22] In a state in which the borrowing constraint binds under the constrained-efficient allocation, we have $\bar{A}_{t+1} = 0$ and hence the households' best response only features a decreasing segment. In that case, the two best responses intersect at $A_{t+1} = 0$.

failure of Ricardian equivalence to increase the economy's net foreign asset position.

The policy is associated with larger gross borrowing than under the constrained-efficient outcome. While this might appear to raise moral hazard concerns, particularly as households anticipate that reserves will be deployed in crises, the welfare improvement holds even when the government cannot commit ex ante to its reserve policy, since we solve for the time-consistent equilibrium. One potential concern, however, is that once households are against their collateral constraint, they cannot borrow more, and so, if households had other forms of borrowing, reserve accumulation could be less effective or even backfire. In Appendix D.4, we extend the model by allowing households to borrow additional funds using an unsecured bond at an interest rate premium and show that our key results remain. Intuitively, while allowing for unsecured debt at a premium induces a cost from reserve accumulation, this cost is second-order starting from the laissez-faire allocation.

Other financial frictions. A natural question is to what extent the results on the desirability of reserves extend or not beyond the form of financial frictions considered in our baseline model. To address this question, Appendix G considers a more general specification of financial frictions. Specifically, we let the cost of borrowing depend on individual and aggregate borrowing, on the government's reserves, and on the current stocks of debt and reserves, without taking a concrete stance on how spreads arise. That is, we assume that households issue debt at a general price schedule $q(b_{t+1}, B_{t+1}, A_{t+1}; B_t, A_t)$. Similar to our model, where households do not internalize how their choice of debt affects the tightness of the collateral constraint for all households, here households do not internalize how their choice of debt affects the spreads faced by other households.

In the appendix, we derive a sufficient condition under which the planner accumulates reserves, and we provide two examples in which reserve accumulation is strictly undesirable. The planner accumulates reserves when the sensitivity of borrowing terms to reserves is sufficiently high relative to the cost of carrying reserves. The condition fails, for instance, when lenders price debt on the economy's net liabilities, or when the rise in gross debt induced by reserve accumulation backfires into less favorable credit conditions in future crises. In these cases, addressing the externality requires a policy that targets gross debt directly, such as capital controls.

Determining whether the sufficient condition holds in practice is ultimately an empirical question. Existing evidence highlights that reserves matter for crisis outcomes in emerging market debtor economies: countries with larger reserve holdings have been less likely to experience crises (Gourinchas and Obstfeld, 2012) and were less affected by the 2008–09 global financial crisis (Frankel and Saravelos, 2012; Bussière, Cheng, Chinn and Lisack, 2015). While this evidence is

only suggestive, it points to reserves improving the terms at which the economy borrows, in line with our model.[23]

4 Quantitative analysis

In this section, we present a quantitative analysis.[24] We organize the results as follows. First, we describe the calibration. Second, we present the policy functions to illustrate the workings of the model. Third, we show that the model can account for the four empirical facts presented in Section 2. Fourth, we present a simple rule for reserve accumulation. Finally, we present various extensions and perform sensitivity analysis.

4.1 Calibration

A period in the model represents a year. The preference parameters for risk aversion and the elasticity of substitution are set to standard values from the literature: $\sigma = 2,\ 1/(1+\eta) = 0.83$. The value for the interest rate is set to 4%, also standard in the literature.[25] For the calibration of the remaining parameters, we use data from Mexico, a common choice in studies of reserve accumulation (e.g., Bianchi et al., 2018) during the period 1980-2015.

To estimate the tradable endowment stochastic process, we use the value added series in the primary and industrial (net of construction) sectors. We assume a first-order autoregressive process for the cyclical component: $\ln y_t^T = \rho^y \ln y_{t-1}^T + \varepsilon_t^y$ with $\varepsilon_t^y \sim N(0, \sigma_y)$, and estimate values of $\rho^y = 0.24$ and $\sigma_y = 0.034$.[26] The value of ω is set to match the share of tradable GDP in the data, which is 33%.[27]

We assume that the financial shock κ_t follows a first-order autoregressive process in logs: $\log(\kappa_t) = (1-\rho^\kappa)\log(\bar{\kappa}) + \rho^\kappa \log(\kappa_{t-1}) + \varepsilon_t^\kappa$ with $\varepsilon_t^\kappa \sim N(0, \sigma_\kappa)$. Unlike income, the financial

[23] More broadly, when financial vulnerability originates from corporate debt rather than household debt, the appropriate policy instruments may differ from those analyzed here, as the channels through which reserves affect borrowing conditions would operate through different sectors of the economy. See Kalemli-Ozcan, Kamil and Villegas-Sanchez (2016).

[24] The competitive equilibrium is solved numerically using time iteration and the optimal policy problem is solved with value function iteration, as in Bianchi (2011).

[25] We consider a stochastic interest rate in Section 4.5.

[26] We use value added data in local currency from Mexico's National Institute of Statistics and Geography (INEGI) for 1980-2015, deflated by sector-specific prices. The cyclical component of the tradable output is estimated using the Hodrick-Prescott filter with a smoothing parameter of 100 on yearly data from 1980 to 2015.

[27] In a non-stochastic version of the model with a level of net foreign asset position equal to $\overline{NFA}$ and tradable and non-tradable output normalized to one, the tradable share of total output is given by the value of ω such that $1/[1 + \frac{1-\omega}{\omega}(1 + r\ \overline{NFA})^{1+\eta}] = 33\%$. Given the mean value of the NFA to be calibrated below, this yields $\omega = 0.33$.

shock is not directly observable. To discipline the process for κ_t, we exploit the fact that the credit constraint holds with equality under the optimal reserve intervention, and follow the approach proposed by Jermann and Quadrini (2012). Namely, taking (17) with equality, we back out a time series for κ_t using the observed sequence of output and debt. Since before the 1994 Tequila crisis, Mexico had very low levels of reserves, we take 1995-2015 as the reference period. We then estimate the aforementioned AR(1) process and obtain $\rho^\kappa = 0.82$, $\bar{\kappa} = 0.46$ and $\sigma_\kappa = 0.11$.

The remaining parameter is the discount factor. We calibrate β so that the average NFA in the economy without government intervention matches the average of Mexico's NFA position.[28] This calibration yields $\beta = 0.94$. A summary of parameter values is provided in Table 2.

Table 2: Parameter Values

	Value	Source/Targets
Interest Rate	$r = 0.04$	Standard value
Risk Aversion	$\sigma = 2$	Standard value
Elasticity of Substitution	$1/(1+\eta) = 0.83$	Standard value
Weight on Tradables in CES	$\omega = 0.33$	Share of T output = 33%
Stochastic structure y^T	$\rho^y = 0.24, \sigma_y = 0.034$	See text
Stochastic structure κ	$\rho^\kappa = 0.82, \bar{\kappa} = 0.46, \sigma_\kappa = 0.11$	See text
Discount Factor	$\beta = 0.94$	Average NFA = −37.0%

4.2 Reserves and gross debt

We start by describing the workings of the model through an analysis of the policy functions for reserve accumulation and debt. In all figures, we compute reserves and debt as a fraction of the unconditional mean of output. We highlight how the reserve intervention differs markedly from a Pigouvian tax intervention.

Policy function for reserves. Figure 4 presents the optimal reserve accumulation policy as a function of the shocks the economy faces and the current value of net liabilities. In panel (a), the amount of reserves is shown as a function of the tradable endowment, for a value of κ one standard deviation below the mean and at two possible values of beginning-of-period net foreign liabilities. In panel (b), the amount of reserves is shown as a function of the financial shock, for a value of y^T one standard deviation below the mean, again for two possible values of net liabilities. In both cases, reserves are reported as a fraction of average GDP.

[28] Although gross positions have increased quite substantially over time, the average NFA is about the same in the periods 1980-1994 and 1995-2015.

Figure 4 shows that the government finds it optimal to hold more reserves in good times, that is, when income is high or when financial conditions are less stringent. The intuition for these results is that when the amount that households can borrow rises (because of either higher y^T or higher κ), the government needs to accumulate more reserves to close the gap between the net amount of borrowing desired by the planner and the borrowing capacity of households. Similarly, when beginning-of-period net foreign liabilities are lower, households are further away from their credit constraint—they want to borrow less and they have more spare borrowing capacity—and the government accumulates more reserves.

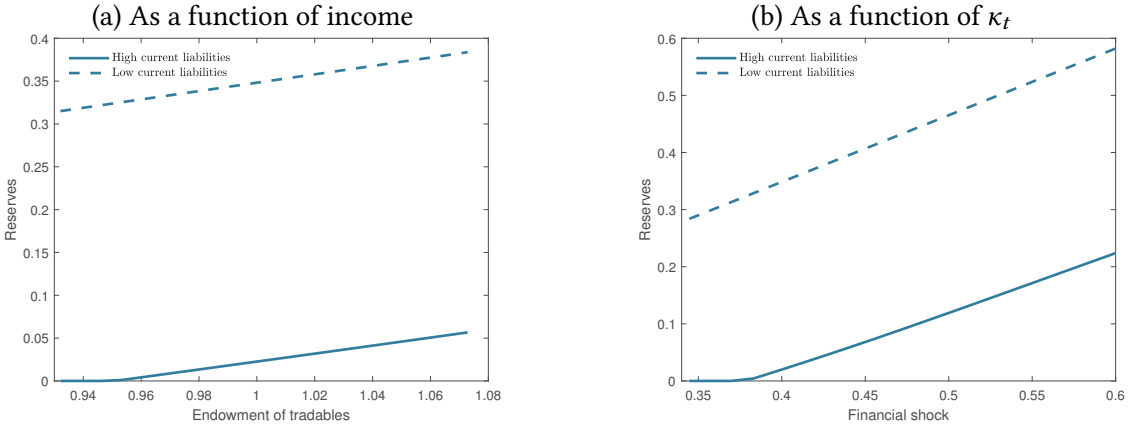


Figure 4: Policy function for reserves

Note: In the left (right) panel, the financial (output) shock is set to one standard deviation below the mean value. Reserves are expressed as a fraction of the unconditional mean of output. Low current liabilities and high current liabilities correspond respectively to initial net foreign liabilities equal to 6 percent and 38 percent of mean output at the ergodic distribution of the constrained efficient economy.

Comparison with taxes on debt. An important fact that motivated our analysis was that countries that rely less on capital controls appear to hold larger amounts of reserves (Fact 4). In our model, reserves and taxes on debt are substitute policies: a government that uses capital controls has no need for reserve accumulation and conversely, a government that accumulates reserves does not need to impose capital controls. It is interesting, however, to contrast the cyclical properties of the optimal reserve intervention policy with those of the optimal capital control policy. Figure 5 again displays policy functions for reserves, but this time together with policy functions for the optimal tax on debt, using the optimal borrowing tax formula of Bianchi (2011).

A common feature of the two classes of policies is that they are passive when the constraint is already binding (both taxes on debt and reserve holdings are zero in this case). However, they differ markedly in terms of their cyclical properties. While reserves tend to *increase* with output,

the tax on debt tends to *decrease* with output. The reason for the distinct cyclical pattern is as follows. When output is low, agents have stronger incentives to borrow, leading to a higher probability of a binding borrowing constraint in the future—hence, calling for a higher tax on debt. By contrast, as we explained above, when output is low, there is a smaller excess borrowing capacity, which calls for a smaller amount of reserves, as can be seen from panel (a) of Figure 5. A similar contrast applies with respect to the financial shock (panel [b]).

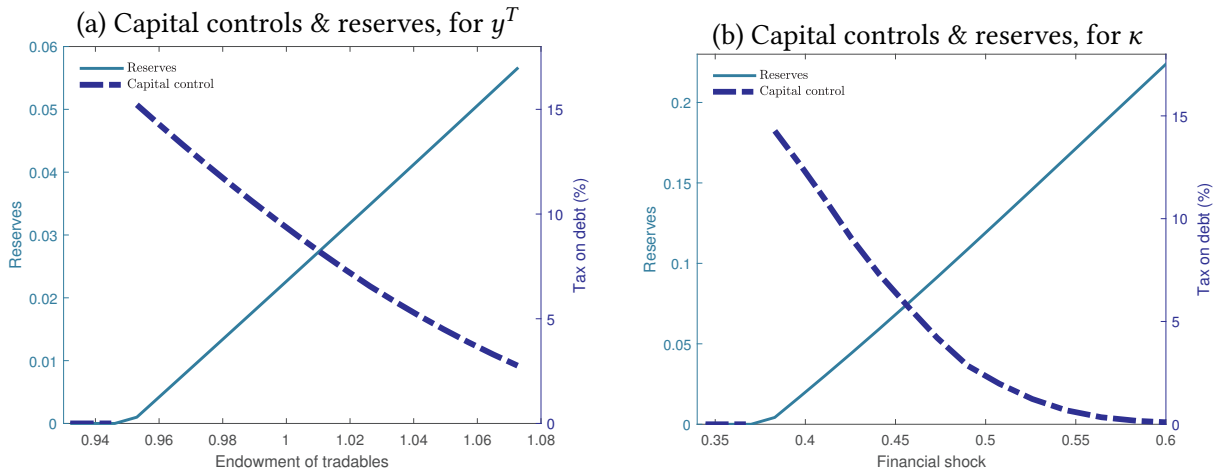


Figure 5: Reserve accumulation vs. capital controls

Note: In panel (a), the financial shock is set to one standard deviation below the mean value. In panel (b), output is set to one standard deviation below the mean value. Reserves are expressed as a fraction of the unconditional mean of output. In both panels, initial net foreign liabilities are 38 percent of the unconditional mean of output.

Policy functions for gross private debt. We now show how the profile of private debt depends on the government intervention. Figure 6, panel (a), shows the law of motion for b' for three economies: (i) laissez-faire, (ii) constrained-efficient, and (iii) foreign reserve intervention.[29] The figure shows that when current net foreign liabilities are high enough, the borrowing constraint binds and all three economies have the same end-of-period debt. For lower liability levels, however, private debt choices differ: the constrained-efficient economy is the one in which the least amount of private debt is accumulated, followed by the laissez-faire economy and the economy with foreign reserve intervention.

The next two panels display the density of debt and credit constraint slackness in the ergodic distribution as a share of mean output. Panel (b) compares private debt distributions under laissez-faire, constrained-efficient, and optimal foreign reserve interventions, highlighting a rightward

[29] By "constrained-efficient," we mean the allocation described in Section 3.4, while by "foreign reserve intervention," we mean the implementation of the constrained-efficient allocation presented in Section 3.5.

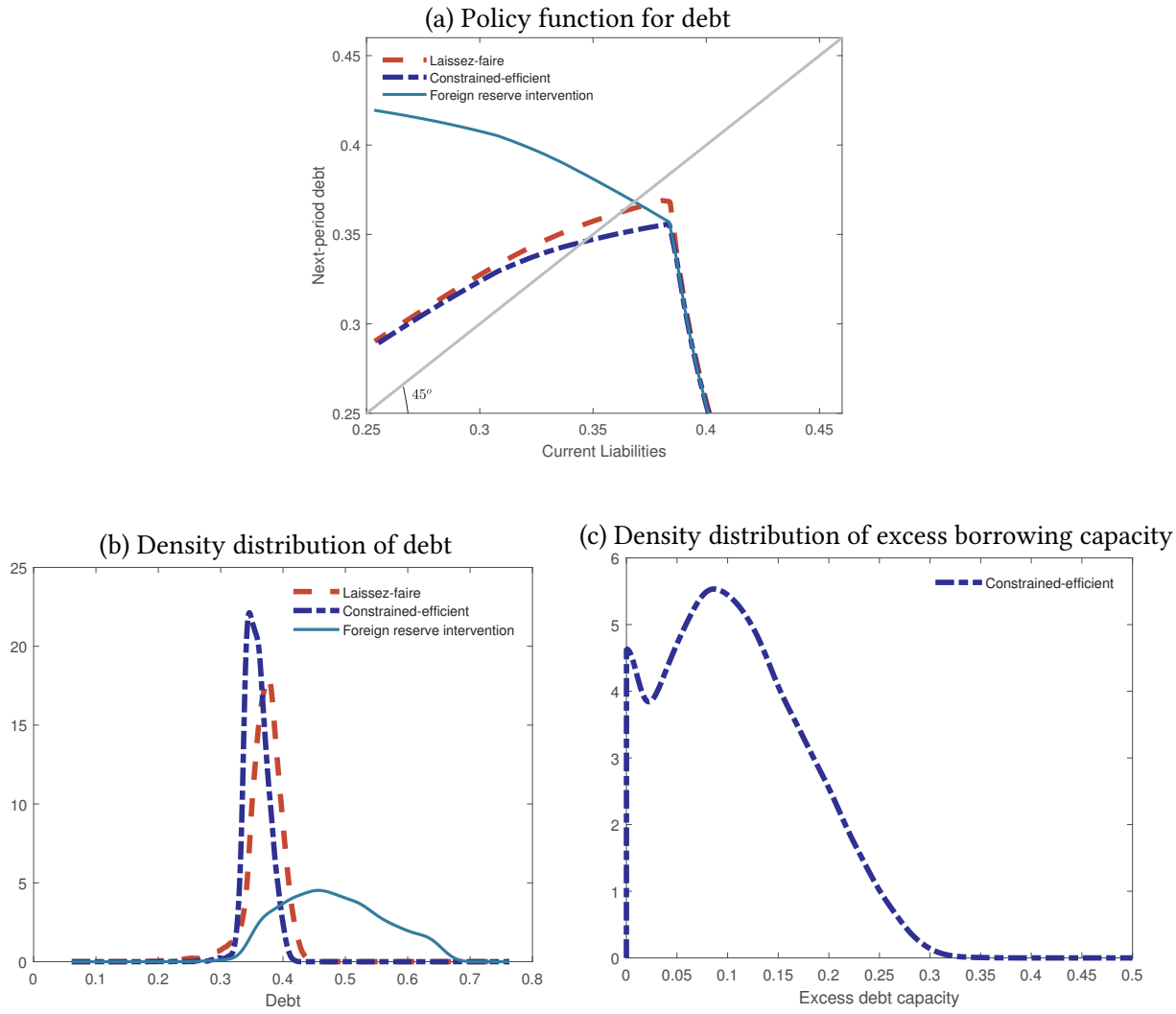


Figure 6: Debt, policy functions and ergodic distributions

Note: In panel (a), the financial and output shocks are set to one standard deviation below their respective mean values. Panel (b) shows the density distribution of debt in the ergodic distribution as a share of the unconditional mean of output. Panel (c) shows the density distribution of the excess borrowing capacity in the ergodic distribution, defined as the difference between the value of collateral and the debt issuance as a share of the unconditional mean of output.

shift in gross private debt under foreign reserve intervention. Panel (c) presents the ergodic distribution of the excess borrowing capacity in the constrained-efficient allocation, defined as the difference between the maximum debt capacity and the debt issuance choice in the constrained-efficient economy. The average slackness at the ergodic distribution is 12% of mean output. Recall that Proposition 1 links the level of reserves to the slackness in the borrowing constraint.

A finding that stands out is that gross private indebtedness is *higher* under the foreign reserve

intervention than in the laissez-faire economy.[30] This result emerges even though the laissez-faire economy features overborrowing relative to the constrained-efficient allocation. In other words, the laissez-faire economy displays a lower NFA position than the economy with the optimal reserve intervention as well as a lower gross debt position. This "underborrowing" result is thus different from the one described by Benigno et al. (2013) in a production economy where the planner issues more debt *and* has a lower NFA position than in the laissez-faire.[31]

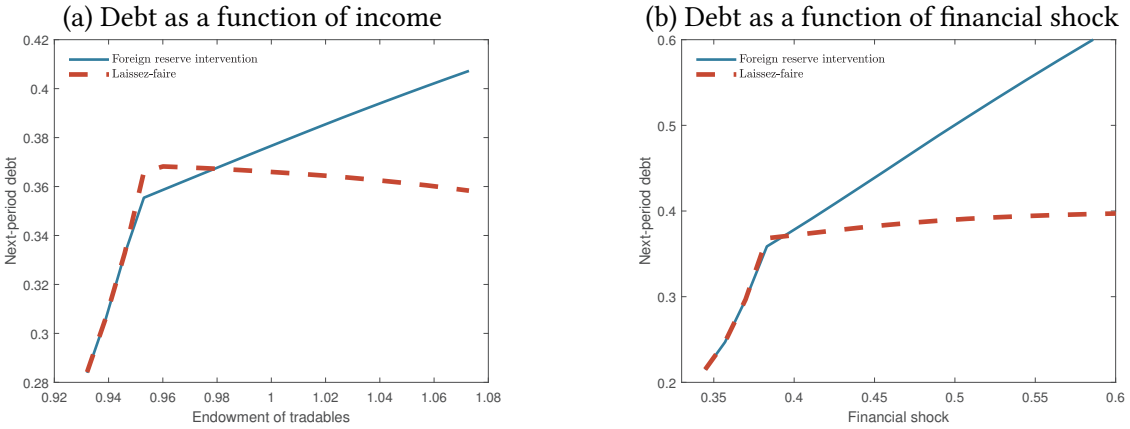


Figure 7: Equilibrium policy function for debt

Note: In panel (a), the financial shock is set to one standard deviation below the mean value. In panel (b), output is set to one standard deviation below the mean value. Debt is expressed as a fraction of the unconditional mean of output. In both panels, initial liabilities are 38 percent of the unconditional mean of output.

Figure 7 further shows how the optimal reserve intervention changes the cyclical properties of private borrowing: the debt policy functions with respect to income and financial conditions are shown in panels (a) and (b), respectively. When income is low, borrowing is increasing in income for both the laissez-faire economy and the economy with the optimal reserve intervention. The reason is that when income is low, the borrowing constraint is binding and higher income helps relax it. When income is high, however, the two economies differ in the cyclical properties of borrowing: while borrowing is countercyclical under laissez-faire, it is procyclical under the optimal reserve intervention. Under laissez-faire, when the credit constraint does not bind, the economy borrows less when income is high, following a permanent income logic. Under the optimal foreign reserve intervention, in contrast, since the excess borrowing capacity is procyclical in the constrained-efficient allocations, the government accumulates more reserves when output

[30] In the state space, this occurs technically for all values of debt except those at which the borrowing constraint is binding under the laissez-faire but not in the constrained-efficient allocation.

[31] Notice, however, that Arce, Bengui and Bianchi (2023) show that regardless of the production structure, the planner always finds it optimal to impose a positive tax on debt and this tax could be larger in configurations with higher borrowing for the planner.

is high, in turn inducing households to take on more debt. On the other hand, panel (b) shows that private borrowing is procyclical with respect to financial conditions in both economies.

Our finding that optimal foreign reserve interventions may lead to higher private indebtedness has implications for empirical studies on credit booms and financial crises. In particular, it stresses the importance of considering official reserve dynamics when determining the role of private credit in predicting financial crises. In our model, the optimal foreign reserve intervention results in higher private indebtedness, yet a lower exposure to financial crises.

Long-run moments. Table 3 displays average debt and reserves (as a fraction of output), together with the long-run probabilities of crisis. The amount of overborrowing in the laissez-faire economy relative to the constrained-efficient allocations is about 1% of GDP, but the amount of debt under the optimal intervention is about 11% of GDP higher than under the laissez-faire. Meanwhile, the optimal average level of reserves is 12% of output. This value is in the range of the recently observed level of reserves, which is around 15% of GDP for Mexico.[32]

It is worth emphasizing that even though the presence of overborrowing is key to justify the reserve accumulation policy, the scope of the intervention needed is not determined by the difference between the constrained-efficient level of borrowing and the level of borrowing in the laissez-faire. Rather, as indicated by (RP), the amount of intervention is determined by the difference between the constrained-efficient level of borrowing and *the economy's borrowing capacity*. This difference can be quite large, especially in states in which financial conditions are loose.

Finally, like optimal capital controls, the optimal reserve accumulation policy substantially mitigates exposure to financial crises. We compute the probability of a financial crisis, defined as an episode where the current account increases by more than two standard deviations above its mean, following the empirical literature. In the laissez-faire economy, the probability of a crisis is 1.8%, which is in the range of the estimated frequency of financial crises for emerging markets (e.g., Calvo et al., 2006). In the simulations, these events are always characterized by binding credit constraints. The probability that the credit constraint binds is 2.5%, implying that nearly 70% of the times that a shock triggers a binding credit constraint, there is a sharp reversal of capital flows. Relative to the laissez-faire economy, following the optimal reserve accumulation policy reduces

[32] As we explained in Section 3, when the credit constraint is currently slack and its probability of being binding in the following period is zero, the reserve policy is indeterminate. While we assume that reserves follow the policy (RP) in our quantitative analysis, assuming that reserves are zero in these circumstances would only change the average level of reserves from 12.2% to 12.0%. The results differ very little because the indeterminacy arises only 2.4% of the time in the simulations.

the probability of financial crises from 1.8% to 0.4%.[33] While the optimal reserve intervention does not fully eliminate the occurrence of crises, it substantially reduces their frequency. As we show in Section 4.4, it also reduces their severity.

Table 3: Long-run moments

	Laissez-faire	Constrained-efficient	Optimal Reserves
Mean Debt	37.2	35.9	48.1
Mean Reserves	-	-	12.2
Crisis probability	1.8	0.4	0.4

4.3 Accounting for the stylized facts

We now assess the model's ability to account for the facts 1–4 outlined in Section 2. To do so, we simulate the model to generate artificial data comparable with the data used in our empirical analysis of Section 2.

Fact 1. First, we examine whether the model can account for the recent increase in reserves while being consistent with the simultaneous rise in private external debt observed in the data. We use our calibration for Mexico. Starting in 2000, we fix the initial gross positions from the data and feed the observed path for income. A simulation of the model also requires a path for the financial shock, which is not observable. Given our interest in the joint trend between reserves and debt, we feed the sequence of financial shocks that delivers the sequence of debt observed in the data. As we show in Figure F.1, the model estimates a secular increase in κ, which is consistent with the rise in overall capital mobility.[34]

Panel (a) of Figure 8 shows that this exercise makes the model predict a significant increase in reserves consistent with the increase observed in the data for Mexico.[35]

The model is therefore able to jointly explain the increase in debt and reserves. Notably, while

[33] The probability of a binding credit constraint is 0.7% in the constrained efficient economy.

[34] The sequence of debt in the data is constructed, analogously to the model, as the sum of net foreign liabilities plus reserves.

[35] Foreign reserves-to-GDP in Mexico increased by a factor of 2.5 between 2000 and 2015, rising from 6% to around 15%. Reserves in our sample of countries in Section 2 increase by a factor of 3. When including China, reserves increase by a factor of 4.

the debt path was targeted in our simulation (see panel [b]), the path of reserves was not.[36]

The model predicts more volatility in reserves than observed in the data. The unconditional standard deviation of reserves-to-output in the model is 6.7 percentage points, compared to 0.9 percentage points for the cyclical component of reserves-to-GDP in Mexico over 1970–2015 (HP filter with smoothing parameter 100).[37] This discrepancy reflects the model's focus on the macroprudential motive, under which reserves serve as a pure shock absorber: the planner fully depletes them whenever the borrowing constraint binds. In particular, our model abstracts from other forces, such as the rollover-crisis motive in Barbosa-Alves, Bianchi and Sosa-Padilla (2024) or the signaling and precautionary considerations that lead central banks to hold more stable buffers. The model therefore substantially overstates cyclical reserve volatility.

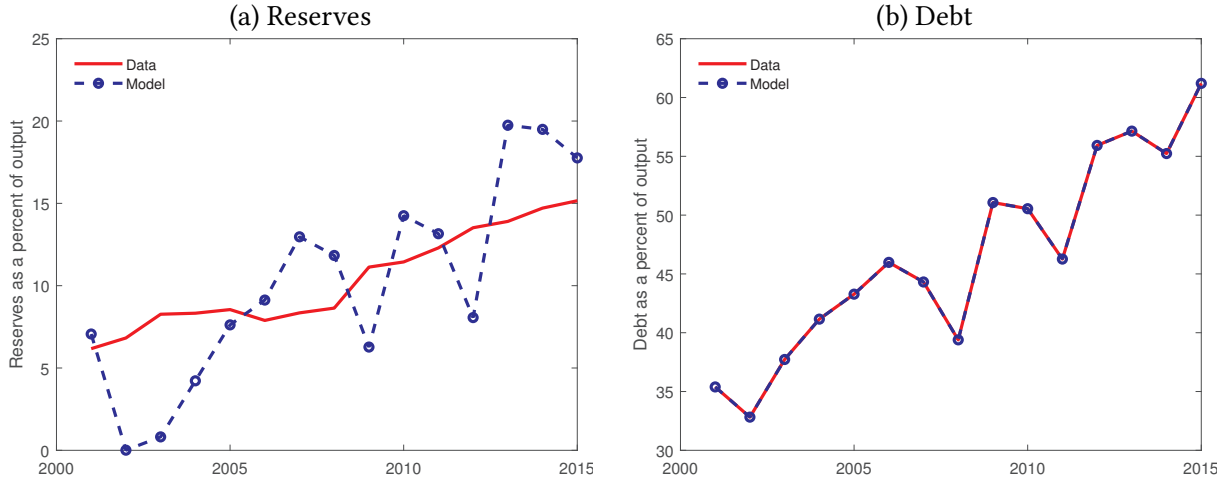


Figure 8: Evolution of reserves and debt, 2001–2015: data and model

Note: Model simulation obtained by feeding observed income shocks and calibrating financial shocks to match the sequence of NFA (excluding reserves) observed in the data.

Fact 2. Second, we argue that our model is also consistent with the positive cross-sectional association between changes in reserves and changes in private external debt observed in the data. To examine this fact through the lens of our model, we proceed in a way analogous to the way

[36] We note that the financial shocks that are reverse-engineered in the procedure are consistent with the log-normal process estimated in the calibration section. In particular, the financial shock is within the 95% confidence interval generated by the estimated process in all but one year from 2001 to 2015 (see Appendix F for details). Notice that for small changes in the constrained-efficient debt-to-output ratio, the change in the reserves-to-output ratio is approximately equal to the change in the value of κ. That is, $\Delta\left(\frac{\mathcal{A}_{t+1}}{y_t^T + p_t^N y^N}\right) \approx \Delta\kappa_t$.

[37] Using linear detrending instead yields a cyclical standard deviation of 1.6 percentage points in the data. Comparing first-differenced series gives 5.4 percentage points in the model versus 1.4 percentage points in the data. The model is stationary by construction, so the natural counterpart of the cyclical volatility in the data is the ergodic standard deviation in the model.

we proceeded in the data: (i) we construct 25 samples of simulations of 10,035 years each, and focus on the last 35 years; (ii) we use this simulated data to construct model counterparts of the changes in reserves-to-output ratio, changes in the external debt-to-output ratio, and the current account-to-output ratio; (iii) we estimate the effect of increases in debt on increases in reserves in a series of panel regressions on the simulated data.

Table 4 reports results from panel regressions on the simulated data where the dependent variable is the net changes in foreign reserves-to-output ratios, and the controls are the changes in the external private debt-to-output ratios and the current account-to-output ratio. Columns 1 to 4 are therefore the model counterparts of columns 1 to 4 of Table 1. As in the data, all the regressions include a constant and all standard errors are clustered at the country (simulated panel) level. Column 1 is a pooled OLS regression, column 2 includes country fixed effects only, column 3 includes time fixed effects only, and column 4 includes both time and country fixed effects. The table shows a positive and significant relation between reserves and private external debt in all specifications, consistent with Fact 2. This association holds both within country panels and across countries.[38] Of course, reserves in the data are also driven by factors other than the macroprudential motive studied in the paper. Hence, the estimated parameters in the data in Table 1 are lower than their model counterparts in Table 4.

Table 4: Changes in Reserves-to-GDP Ratios on changes in Private External Debt-to-GDP Ratios

	(1) Reserves	(2) Reserves	(3) Reserves	(4) Reserves
Private External Debt	1.107^{***}	1.108^{***}	1.104^{***}	1.105^{***}
	(0.0210)	(0.0211)	(0.0220)	(0.0221)
Current Account	1.043^{***}	1.050^{***}	1.044^{***}	1.052^{***}
	(0.199)	(0.203)	(0.201)	(0.206)
Observations	875	875	875	875
Countries	25	25	25	25
Fixed Effects	No	Country	Time	Country+Time
Clustered SE	Yes	Yes	Yes	Yes

Note: Standard errors clustered at the country level in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Results based on 25 samples of simulations of 35 years each, at their respective ergodic distribution, with each observation representing a country-year.

[38] In the appendix, we show that we also obtain a positive and significant relation when we apply panel FGLS to the simulated data.

Fact 3. Third, we show that our model generates time-series correlations between the changes in reserves, private external debt, and output. For each of our 10,000 samples, we compute the time-series of first differences of reserves, private debt, and output.[39] We then calculate the correlation between the reserves and output series and between the private debt and output series. We obtain positive correlations in line with the data, respectively 0.65 and 0.61.

Fact 4. Fourth, we address the correlation between the accumulation of international reserves and capital account openness. So far, we have only considered implementations with either taxes or reserves. However, we can extend our analysis to address Fact 4. We postulate that in the background there is a maximum tax rate $\bar{\tau}$ on borrowing that governments can or are willing to impose, either because of a fear of leakages or because of other unintended consequences. We assume that this maximum tax rate is heterogeneous across countries and draw for each of 10,000 simulations a different $\bar{\tau}$ from a uniform distribution between zero and τ^{max}, where we take τ^{max} to be the largest tax necessary to implement the constrained-efficient allocation in the ergodic distribution. For each of these samples, we consider a government using a mix of capital controls and reserves. We assume that the government implements the constrained-efficient allocation using taxes on borrowing if the optimal tax rate is below the drawn maximum tax rate $\bar{\tau}$ while if the maximum tax is binding, the government sets the maximum tax rate and resorts to reserve accumulation to close the gap.

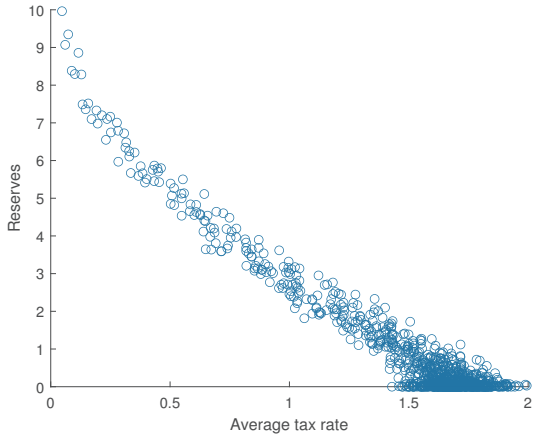


Figure 9: Average reserves and taxes in simulated economies

Note: We simulate 10,000 samples for 30 years each, with each dot representing a sample. The x-axis measures the average tax rate over the 30 periods. The y-axis measures the average level of reserves as a percent of output over 30 periods.

[39] We use the log of the private debt and output series, but not of the reserve series since there are several occurrences of zero reserves in the samples.

Figure 9 plots the average tax rate and reserves for each sample over 30 years. We find that in samples where average reserves are high, taxes on private borrowing are low. This negative correlation between reserves and traditional capital control policy is consistent with our Fact 4.

4.4 A simple rule for reserve accumulation

Motivated by practical policy considerations, we analyze a version of the model in which the government follows a simple policy rule for the accumulation of international reserves.[40]

The simple rule we propose approximates the optimal reserve accumulation by a linear function of state variables, including the NFA position:

$$A_{t+1}^{SR} = \max\{\beta_0 + \beta_1 y_t^T + \beta_2 \kappa_t + \beta_3 (A_t - b_t), 0\},$$

where the $\{\beta_i\}_{i=0}^3$ are constant parameters. We estimate these coefficients by maximizing the unconditional welfare gains from moving from the laissez-faire economy to the economy with the optimal simple rule.[41]

The optimization yields the following coefficients:

$$\beta_0 = -0.34,\ \beta_1 = 0.45,\ \beta_2 = 0.68,\ \beta_3 = 0.29.$$

The estimated rule implies a reserve accumulation policy that is increasing in the current NFA position, income, and the financial shock. The rule hence inherits the same basic qualitative properties as the optimal state-contingent policy but differs in that it is a simple linear rule.

The financial stability gains from adopting the simple rule can be illustrated by conducting the following event analysis. First, we simulate the laissez-faire economy for a large number of periods, identify all the financial crisis episodes, and construct 7-year window events centered around the financial crisis episodes. Second, we take the average of key variables across the window period for the laissez-faire economy. Third, we feed the initial state and shock sequence

[40] Different from the feedback rule (27), the simple rule we consider does not require the government to know the constrained-efficient level of borrowing. It is possible to map this simple rule into a feedback rule where we replace the constrained-efficient level of borrowing in (27) with a target that depends on observables.

[41] Numerically, we proceed by first running an OLS regression of the optimal level of reserves at the ergodic distribution on the exogenous state variables and the current NFA position in the economy with the optimal reserve intervention. We restrict the sample to observations where next period's level of international reserves is strictly positive ($A_{t+1} > 0$). Then, we construct a grid for each of the four parameters $\{\beta_i\}_{i=0}^3$ centered around the OLS estimates. Given three values for each parameter and a total of four parameters, we have eighty-one possible combinations. We select the combination that gives the highest welfare gains. We repeat the process by centering the new grids on this combination. We iterate until we cannot increase the welfare gains by selecting any other point in the grid different from our initial guess.

for each event from the laissez-faire economy to the policy functions of the economies with the simple rule and with the optimal state-contingent reserve policy, again taking the average of key variables.

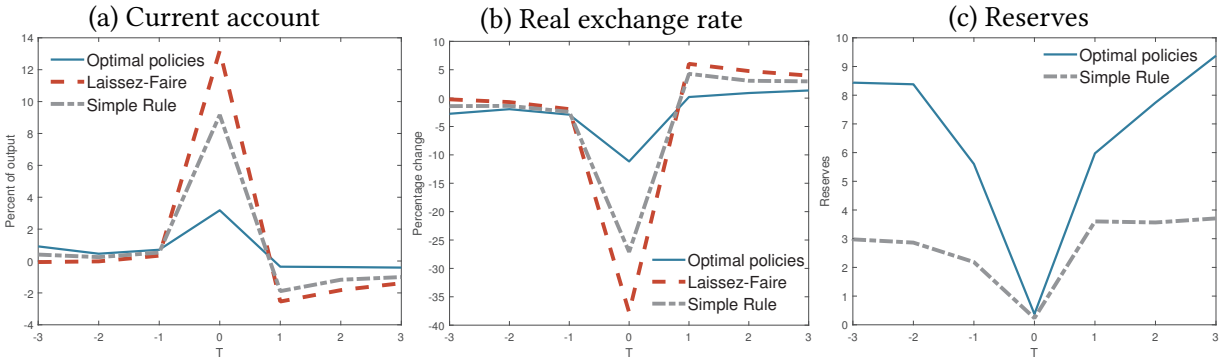


Figure 10: Financial crisis event analysis under optimal policies, laissez-faire, and a simple rule

Note: We conduct an event analysis around a crisis event under laissez-faire policies (dashed red), optimal reserves accumulation (solid blue), and under a simple reserve accumulation policy (dash-dotted gray). A crisis event is defined as an increase in the current account of more than two standard deviations above the mean.

The dynamics of the event analysis are shown in Figure 10. The paths for the current account and the real exchange rate are shown in panels (a) and (b), respectively, comparing outcomes in the laissez-faire economy, the economy with the optimal state-contingent reserve policy and the economy with the simple rule. The figure shows how, in a crisis, the laissez-faire economy experiences a large current account reversal of about 13% of GDP while the real exchange rate depreciates by close to 40%. These magnitudes are in line with empirical regularities of sudden stops (see, e.g., Calvo et al., 2006). The optimal reserve policy is successful at mitigating the severity of sudden stops as reflected in a more modest current account reversal of about 3% of GDP and a real exchange rate depreciation of 10%. The optimal use of reserves, therefore, reduces capital outflows by 10% of GDP and reduces the exchange rate depreciation by 30 percentage points.[42] The simple rule also delivers significant gains, reducing the current account reversal by about 5% of GDP and the real exchange rate depreciation by about 10 percentage points. In terms of welfare, we find that the simple policy rule delivers 12% of the total welfare gains achieved by optimal state-contingent reserve policy.

Panel (c) compares the path for reserves under the optimal policy with its counterpart under the simple rule. As it turns out, the simple rule prescribes less reserve accumulation than the fully

[42]We note that while the model is purely real, it would be relatively straightforward to extend it to a monetary model where the government defends the *nominal* exchange rate. For example, if the government followed an inflation-targeting policy, it would prevent a large nominal exchange rate depreciation to keep inflation on target during a sudden stop.

optimal policy. The inability to conduct a perfectly state-contingent policy leads the government to err on the side of lower reserves. The intuition for this result is that excessive reserve accumulation relative to the optimal may have the effect of excessively tightening households' borrowing constraints. In both cases, reserves fall to a value close to zero around crises, in line with our result on reserve depletion following from Proposition 1 and the empirical evidence that reserves fall sharply during crises (Broner et al., 2013).[43]

4.5 Sensitivity and Model Extensions

In this section, we present several extensions of our model.

Production economy. We consider an extension of the model with production. This extension is important in light of the findings that endogenous production may call for ex-post stabilization policies and affect the efficient amount of borrowing (Benigno et al., 2013). As detailed in Appendix D.3, the extended framework includes labor reallocation across sectors, with more labor shifting to the non-tradable sector during expansions—consistent with the findings of Benigno, Converse and Fornaro (2015). A quantitative analysis of the production economy model with an ex-post stabilization policy shows that the scope for reserve accumulation remains broadly in line with that of our baseline endowment economy model.[44] Implementing the constrained-efficient allocation with reserves and a labor tax in the non-tradable sector results in a reduction in the frequency and severity of financial crises comparable to those obtained in the endowment economy model, as indicated in Table 5. The average reserve level, at 8.7% of output, is somewhat lower than, but of the same order of magnitude as, the baseline value.

Liquidity role of reserves. We argued above that in a crisis, it is optimal for the government to fully deplete its reserves. To further highlight the liquidity role of reserves in mitigating the severity of financial crises, we consider a scenario in which the government suboptimally keeps a fraction of reserves. More precisely, we construct an event analysis, as in Section 4.4, but assume that at the time of the crisis, the government unexpectedly deviates from the optimal reserve policy for one period and keeps 25% of its current level of international reserves. Figure 11 compares the dynamics of crises under this suboptimal policy with those associated with the

[43] The reason why reserves do not exactly fall to zero in the Figure is that for some shock sequences that lead to crises in the laissez-faire economy, the ex-ante reserve accumulation succeeds at averting a crisis altogether.

[44] Our calibration assumes that the productivity of tradables follows a log-normal AR(1) of the form: $\ln A_t^T = \rho^A \ln A_{t-1}^T + \varepsilon_t^A$ with $\varepsilon_t^A \sim N(0, \sigma_A)$. The persistence parameter $\rho^A = 0.24$ is equal to the persistence of the endowment of tradables in the original model. The volatility $\sigma_A = 0.017$ is chosen to ensure that the standard deviation of tradable output as a share of output at the ergodic distribution coincides with its counterpart in the endowment economy.

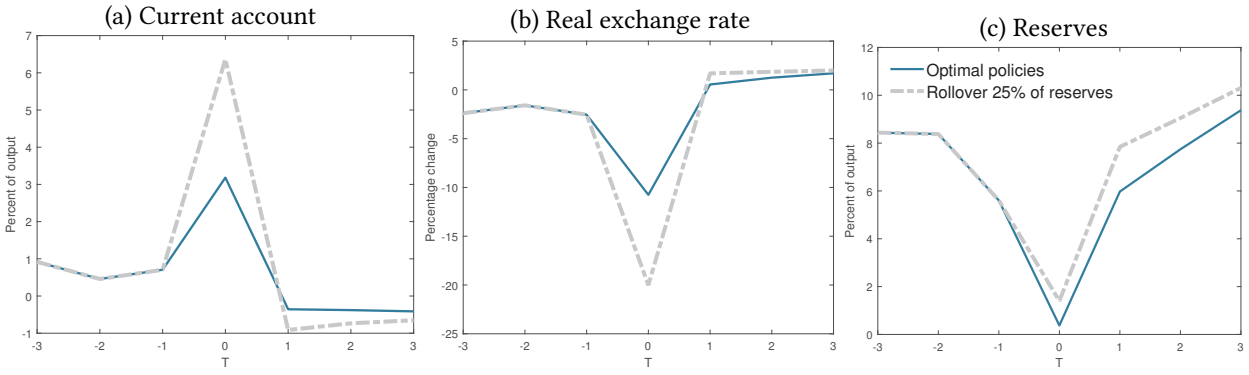


Figure 11: Financial crises event analysis when reserves are not fully depleted.

Note: We conduct an event analysis around a crisis event under optimal reserves accumulation policies (solid blue), and under a reserve accumulation policy that preserves 25% of reserves (dash-dotted gray). A crisis event is defined as an increase in the current account of more than two standard deviations above the mean.

optimal intervention. The experiment points to a strong liquidity role of reserves, as the depletion of an additional 1 percentage point (of GDP) of reserves reduces the current account reversal by over 3 percentage points (of GDP) and the real exchange rate depreciation by nearly 10 percentage points.

Interest rate shocks. We also consider shocks to the risk-free interest rate R_t to capture fluctuations in US monetary policy. Naturally, when interest rates are low, the model predicts an increase in consumption and a reduction in the desired NFA position. However, the effects on gross positions are more subtle. Reserves, in particular, respond to the gap between the economy's borrowing capacity and the constrained-efficient level of borrowing, as revealed by (RP). Keeping all other parameters at their calibrated values and using the interest rate process estimated by Bianchi, Liu and Mendoza (2016), we obtain quantitative results very similar to those obtained in our baseline model with a constant risk-free rate (see Table 5). In particular, the average level of reserves is unchanged at 12.2% of output and the reduction in the frequency and severity of financial crises remains comparable.

Table 5: Long-run moments in extended models

	Baseline		Production economy		Stochastic R	
	Laissez-faire	Optimal	Laissez-faire	Optimal	Laissez-faire	Optimal
Reserves-GDP	-	12.2	-	8.7	-	12.2
Crisis probability	1.8	0.4	1.9	0.3	1.8	0.3
CA reversal	13.2	2.8	6.6	2.6	13.0	2.6

5 Conclusions

This paper has articulated a macroprudential theory of foreign reserve accumulation in which reserves provide a liquidity value not internalized by private agents. Absent government intervention, households overborrow relative to the constrained-efficient allocation. When the government accumulates reserves, households respond by borrowing more, but the failure of Ricardian equivalence implies that reserves ultimately raise the NFA position. The model is consistent with key aspects of the interaction between private and public capital flows observed in the data. It can account for the increase in reserves and private debt over the past three decades, the positive association between these variables in the cross-section and the time series, and the observation that countries with more open financial accounts accumulate more international reserves.

Our analysis provides a benchmark where reserve accumulation can help reduce financial vulnerability and improve welfare. Important practical considerations on the choice between reserve accumulation and capital controls include the degree of leakages in financial regulation (see Bengui and Bianchi, 2022), the political autonomy of the central bank when it comes to utilizing reserves, and the distortionary effects of taxation. Future research could investigate reserve accumulation in models of financial crises with aggregate demand externalities (Schmitt-Grohé and Uribe, 2016, Ottonello, 2015, Coulibaly, 2023), corporate credit and investment (Kalemli-Ozcan et al., 2016), banking crises (Amador and Bianchi, 2024), or foreign intermediaries with limited capital, as in Gabaix and Maggiori (2015).[45]

[45] Recent work by Davis, Devereux and Yu (2023) advances this line of work.

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Online Appendix to "A Macroprudential Theory of Foreign Reserve Accumulation"

A Appendix to Empirical Analysis

A.1 Data Sources

The sources for the data used in the paper are as follows:

Gross Domestic Product (GDP): *updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007)*, GDP in current USD converted from domestic currency using the period-average exchange rate, for all countries from 1980 to 2015, and for Mexico from 1970 to 2015.

International reserves: *updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007)*, "FX reserves" in millions of current USD. Does not include gold holdings, for all countries from 1980 to 2015, and for Mexico from 1970 to 2015.

Net Foreign Assets (NFA): *updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007)*, NFA in millions of current USD, for all countries from 1980 to 2015, and for Mexico from 1970 to 2015.

Private debt: *World Bank 2018. International Debt Statistics*, External debt stocks, private nonguaranteed (PNG) (DOD, current USD) from 1980 to 2015.

Public debt: *World Bank 2018. International Debt Statistics*, External debt stocks, public and publicly guaranteed (PPG) (DOD, current USD) from 1980 to 2015.

Real GDP growth: *IMF 2018. The World Economic Outlook (WEO) database*, Gross domestic product, constant prices in Percent change, from 1980 to 2015.

Capital Account openness: *Chinn and Ito (2008)*. Index of capital account openness. Annualized using data from 1980Q1-2015Q4.

Current account (CA): *updated and extended version of dataset constructed by Lane and Milesi-Ferretti (2007)*, CA in millions of current USD, for Mexico from 1970 to 2015.

Tradable share of GDP: *National Institute of Statistics and Geography (INEGI)*. We first compute the annual sectoral value added in the primary and secondary sector as a share

of total value added from 1980 to 2015. Since the secondary sector includes construction we compute the average share in the disaggregated series, available from 1993-2015. We estimate the share of the construction sector in total value added, 8% with a standard deviation of 0.5%. We subtract this average share from our first measure of tradable value added.

A.2 Sample of Countries

The sample of countries corresponds to "Middle-Income Countries". We arrive at our sample as follows. We start by considering the universe of all countries included in the International Debt Statistics dataset and exclude those listed as "Advanced economies" by the IMF and "Low income countries" by the World Bank. To have a balanced panel from 1980-2015 we keep only countries that have positive values of private debt in the International Debt Statistics and that do not have missing values for the Chinn-Ito index of capital account openness or in the WEO and Lane and Milesi-Ferretti databases. The requirement of a balanced panel subtracts 40 countries from the sample. Finally, we also exclude countries that record net foreign asset positions outside the interval $[-150\%, 150\%]$ of GDP at any point from 1980-2015: Mauritius, Cote d'Ivoire, Jamaica, Papua New Guinea, and Paraguay.

The final list includes the following 25 countries: Argentina, Bolivia, Brazil, Cameroon, Colombia, Costa Rica, Ecuador, Egypt, El Salvador, Guatemala, Honduras, India, Indonesia, Malaysia, Mexico, Morocco, Nigeria, Pakistan, Peru, Philippines, Sri Lanka, Thailand, Tunisia, Turkey, and Venezuela.

A.3 Robustness of the Empirical Results

This section presents additional regression results regarding the empirical facts presented in Section 2. As a reminder, the econometric model that underlies Fact 2, presented in Table 1, is:

$$\Delta a_{it} = \alpha_i + \alpha_t + \gamma x_{it} + \beta \Delta b_{it} + \varepsilon_{it}, \tag{A.1}$$

where Δa_{it} and Δb_{it} denote the yearly changes in the foreign reserves-to-GDP ratio and private external debt-to-GDP ratio for country i, and x_{it} denotes the current account-to-GDP ratio for country i. Since the dependent variable and the explanatory variables are both scaled by GDP, one might be worried that the estimates may suffer from endogeneity bias. To address this issue, we run the same regression but with nominal reserves and private external debt and with real reserves and private external debt (deflating the nominal values by the US Consumer Price Index).

The results are presented in Tables A.1 and A.2. As when variables are scaled by GDP, we find a significantly positive association between private external debt and reserves.

Table A.1: Net changes in Reserves on changes in Private External Debt in millions of USD (nominal)

	(1) Reserves	(2) Reserves	(3) Reserves	(4) Reserves
Private External Debt	0.731***	0.683***	0.671***	0.616**
	(0.178)	(0.176)	(0.180)	(0.179)
Current Account	0.129**	0.187**	0.0933*	0.148**
	(0.0441)	(0.0522)	(0.0407)	(0.0496)
Observations	875	875	875	875
Countries	25	25	25	25
Fixed Effects	No	Country	Time	Time+Country

Note: Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table A.2: Net changes in Reserves on changes in Private External Debt in millions of USD (real)

	(1) Reserves	(2) Reserves	(3) Reserves	(4) Reserves
Private External Debt	0.582***	0.535***	0.540***	0.487**
	(0.150)	(0.143)	(0.148)	(0.141)
Current Account	0.00124**	0.00219***	0.000874*	0.00180**
	(0.000444)	(0.000568)	(0.000420)	(0.000560)
Observations	875	875	875	875
Countries	25	25	25	25
Fixed Effects	No	Country	Time	Time+Country

Note: Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Next, we verify the robustness of the estimation technique. This time, however, the parameter of interest, β, is estimated using panel Feasible Generalized Least Squares (FGLS). The regression results are presented in Table A.3. As before, all the regressions include a constant and control for the current account-to-GDP ratio. Moreover, to obtain accurate standard errors, we conduct preliminary tests on the error terms, ε_{it}. We find that the error structure is heteroskedastic, serially autocorrelated within panels, and contemporaneously correlated across panels. Our parameter estimates in Table A.3 are therefore robust to heteroskedasticity and cross-sectional correlation and assume a panel-specific autoregressive process of order one for the error terms. The coefficient of

interest is again positive and statistically significant at the 1 percent significance level, confirming a robust statistical association between changes in private external debt and changes in reserves.

Table A.3: Net changes in Reserves-to-GDP Ratios on changes in Private External Debt-to-GDP Ratios

	(1) Reserves	(2) Reserves	(3) Reserves	(4) Reserves
Private External Debt	0.253^{***}	0.268^{***}	0.250^{***}	0.249^{***}
	(0.0150)	(0.0143)	(0.0150)	(0.0146)
Current Account	0.170^{***}	0.246^{***}	0.159^{***}	0.244^{***}
	(0.0119)	(0.0130)	(0.0124)	(0.0138)
Observations	875	875	875	875
Countries	25	25	25	25
Fixed Effects	No	Country	Time	Time+Country

Note: Standard errors in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

We next verify that this positive association also holds in the simulated data. As in Section 4.3, we use the calibrated model to construct 25 samples of simulations of 10,035 years each and focus on the last 35 years. We use the model counterparts of the changes in the reserves-to-output ratio, changes in the external debt-to-output ratio, and the current account-to-output ratio in FGLS panel regressions. The parameter estimates are shown in Table A.4. The estimates and their respective standard errors are computed assuming the same error structure as in the empirical data. We obtain positive and significant parameters for all model specifications.

In Table A.5 we verify the robustness of Fact 3 and Fact 4 using a pooled OLS regression approach with clustered standard errors at the country level. In column 1 (column 2) we regress the growth rate in foreign reserves (private debt) on a constant and the real GDP growth rate. Once again we find that reserves and debt accumulation are procyclical and that this relation is statistically significant. In column 3 we regress the ratio of reserves-to-GDP on a constant and the Chinn and Ito (2008) index of capital account openness for our panel of countries. The positive association between reserves and capital account openness holds. To make sure that this result is not driven by outliers, in column 4 we drop from the sample all the observations from Thailand and Malaysia, and verify that the relation is still positive and significant.

Table A.4: Net changes in Reserves-to-output Ratios on changes in Private External Debt-to-output Ratios in the simulated data

	(1) Reserves	(2) Reserves	(3) Reserves	(4) Reserves
Private External Debt	1.107***	1.107***	1.108***	1.108***
	(0.00514)	(0.00514)	(0.00586)	(0.00586)
Current Account	1.114***	1.114***	1.115***	1.115***
	(0.0229)	(0.0229)	(0.0254)	(0.0254)
Observations	875	875	875	875
Countries	25	25	25	25
Fixed Effects	No	Country	Time	Country+Time
Clustered SE	Yes	Yes	Yes	Yes

Note: Standard errors clustered at the country level in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Based on 25 samples of simulations of 35 years each, at their respective ergodic distribution, with each observation representing a country-year.

Table A.5: Robustness of Fact 3 and Fact 4

	Fact 3		Fact 4	
	(1) Growth in Reserves	(2) Growth in Private External Debt	(3) Reserves	(4) Reserves
Real GDP Growth	1.098**	0.907***		
	(0.474)	(0.265)		
Index of Capital Account Openness			4.925*	7.776***
			(2.991)	(1.921)
Observations	865	865	875	805
Countries	25	25	25	23
Clustered SE	Yes	Yes	Yes	Yes

Note: Standard errors clustered at the country level in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

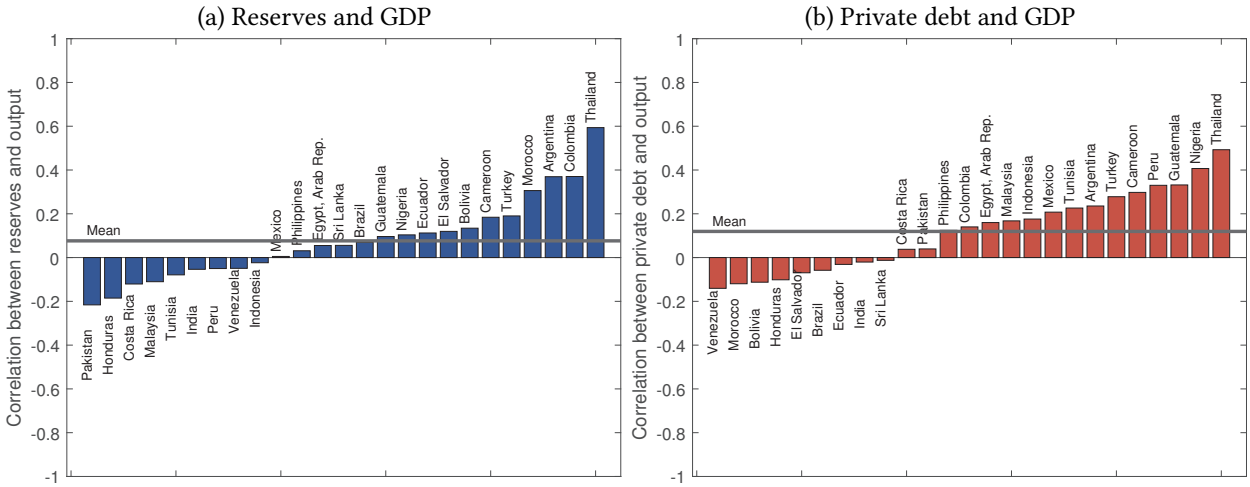


Figure A.1: Fact 3: Time-series correlations

Note : Correlation between the growth rates of real GDP and growth rate of reserves (panel a), and growth rate of real GDP and growth rate of private debt (panel b)

B Proofs

In this section, we present the proofs not presented in the main text.

B.1 Implementation for more general parameterizations

We provide here a proposition analogous to Proposition 1, but without using Cobb-Douglas preferences and Assumption 1.

Proposition B1. *Consider the solution to the constrained-efficient planning problem* $\{c_t^{T\star}, b_{t+1}^{\star}, p_t^{N\star}\}$. *Assume that the solution satisfies the following condition*

$$\mu_t^{\star}(\Psi_t^{\star} - 1) \leq \beta R_t \mathbb{E}_t \mu_{t+1}^{\star} \Psi_{t+1}^{\star} \tag{B.1}$$

for all t *where* Ψ_t *is defined in* (16) *and* $\mu_t^{\star}$ *is given by* (21). *Then, given initial conditions* (b_0, A_0) *such that* $b_0^{\star} = b_0 - A_0$, *the decentralized equilibrium with a consumption allocation* $\{c_t^{T\star}\}$ *can be implemented if the government follows the reserve policy* $\{A_{t+1}\}$ *given by* (RP).

Proof. The proof follows the same steps as in Proposition 1, but uses (B.1) to show that given (23), we have that $\mu_t \geq 0$ is satisfied in the decentralized equilibrium. Notice that when Assumption 1 holds, condition (B.1) is trivially satisfied. ∎

B.2 Proof of Proposition 2

Proof. By construction, the feedback rule (27) guarantees that the economy's net foreign asset (NFA) position always coincides with its counterpart in the constrained efficient allocation $A_{t+1} - b_{t+1} = -b_{t+1}^{\star}$. Substituting this equality into the economy's resource constraint (10) shows that the constrained efficient level of consumption is also achieved. Following the same steps as in the proof of Proposition 1, conditions (21), (22), and (23) must hold. When $\mu_t^{\star} > 0$ or $\mu_{t+1}^{\star} > 0$ in at least one successor state, (23) indicates that $\mu_t > 0$, so that $b_{t+1}/R_t = \kappa_t(y_t^T + p_t^{N\star} y^N)$ and $A_{t+1} = R_t \kappa_t (y_t^T + p_t^{N\star} y^N) - b_{t+1}^{\star} \geq 0$. Meanwhile, when $\mu_t^{\star} = 0$ and $\mu_{t+1}^{\star} = 0$ in every successor state, (23) indicates that $\mu_t = 0$, so that $b_{t+1} = b_{t+1}^{\star}$ is optimal and $A_{t+1} = 0$. Either way, the private borrowing choice is optimal and reserves are non-negative. ∎

C Optimal reserve accumulation policy

We consider the problem of the government that chooses a state-contingent sequence $\{A_{t+1}\}_{t=0}^{\infty}$ to maximize welfare in the competitive equilibrium. Let us use $U(c^T, c^N)$ to denote $u(c(c^T, c^N))$. The problem consists of solving:

Problem 1 (Optimal Policy)**.**

$$\max_{b_{t+1}, A_{t+1}, c_t^T, \mu_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t^T, y^N) \tag{C.1}$$

subject to

$$b_t + c_t^T = y^T + A_t + \frac{b_{t+1} - A_{t+1}}{R_t}, \qquad (\hat{\lambda}_t) \tag{C.2}$$

$$\frac{b_{t+1}}{R_t} \leq \kappa \left(y_t^T + \frac{1-\omega}{\omega} \left(\frac{c_t^T}{y^N} \right)^{\eta+1} y^N \right) \qquad (\hat{\mu}_t) \tag{C.3}$$

$$A_{t+1} \geq 0, \qquad (\hat{\zeta}_t) \tag{C.4}$$

$$u_T(c_t^T, y^N) = \beta R_t \mathbb{E}_t u_T(c_{t+1}^T, y^N) + \mu_t, \qquad (\hat{\xi}_t) \tag{C.5}$$

$$0 = \mu_t \left[\frac{b_{t+1}}{R_t} - \kappa \left(y^T + \frac{1-\omega}{\omega} \left(\frac{c_t^T}{y^N} \right)^{\eta+1} y^N \right) \right], \qquad (\hat{\chi}_t) \tag{C.6}$$

$$\mu_t \geq 0. \qquad (\hat{\nu}_t) \tag{C.7}$$

where $\hat{\lambda}_t, \hat{\mu}_t, \hat{\xi}_t, \hat{\zeta}_t, \hat{\chi}_t$ and $\hat{\nu}_t$ are the multipliers associated with the constraints (C.2)-(C.7).

The government's first-order conditions for b_{t+1}, A_{t+1}, c_t^T and μ_t are given by

$$b_{t+1}: \quad \frac{\hat{\lambda}_t}{R_t} + \frac{\hat{\chi}_t \mu_t}{R_t} = \beta \mathbb{E}_t \hat{\lambda}_{t+1} + \frac{\hat{\mu}_t}{R_t}, \tag{C.8}$$

$$A_{t+1}: \quad \frac{\hat{\lambda}_t}{R_t} = \beta \mathbb{E}_t \hat{\lambda}_{t+1} + \hat{\zeta}_t, \tag{C.9}$$

$$c_t^T: \quad \hat{\lambda}_t = u_T(t) + \hat{\xi}_t u_{TT}(c_t^T, y^N) - \hat{\xi}_{t-1} R_{t-1} u_{TT}(c_t^T, y^N) + \Psi_t(\hat{\mu}_t - \hat{\chi}_t \mu_t), \tag{C.10}$$

$$\mu_t: \quad \hat{\xi}_t = \hat{\chi}_t \left[\frac{b_{t+1}}{R_t} - \kappa \left(y^T + \frac{1-\omega}{\omega} \left(\frac{c_t^T}{y^N} \right)^{\eta+1} y^N \right) \right] + \hat{\nu}_t, \tag{C.11}$$

and complementary slackness conditions

$$0 = \hat{\mu}_t \left[\frac{b_{t+1}}{R_t} - \kappa \left(y^T + \frac{1-\omega}{\omega} \left(\frac{c_t^T}{y^N} \right)^{\eta+1} y^N \right) \right], \tag{C.12}$$

$$0 = \hat{\zeta}_t A_{t+1}. \tag{C.13}$$

Combining conditions (C.9) and (C.10) reveals that the government faces different marginal benefits from accumulating reserves. In a state in which (C.6)-(C.7) are not binding—as will be indeed the case at the optimum—and assuming that the constraints for reserves and borrowing are currently slack, we have that

$$u_T(t) = \beta R_t \mathbb{E}_t \Big[u_T(t+1) + \hat{\mu}_{t+1} \Psi_{t+1} + \hat{\xi}_{t+1} u_{TT}(c_{t+1}^T, y^N) - \hat{\xi}_t R_t u_{TT}(c_{t+1}^T, y^N) \Big] + \hat{\xi}_{t-1} R_{t-1} u_{TT}(c_t^T, y^N) - \hat{\xi}_t u_{TT}(c_t^T, y^N) \tag{C.14}$$

There are two new terms in the planner's Euler equation for reserves relative to the household version. First, there is the pecuniary externality, captured by the $\hat{\mu}_{t+1}\Psi_{t+1}$ term, which reflects that the planner internalizes that having more reserves in a future state with a binding borrowing constraint has positive general equilibrium effects. Second, there is an incentive term that captures how households respond to government policy. Chiefly important for this effect is that the planner is subject to households' borrowing Euler equation as an implementability constraint and that because of the overborrowing externality, the Lagrange multiplier is non-negative. When the government accumulates reserves, this lowers at the margin current consumption and increases future consumption. These effects tighten today's implementability constraint and relax next period's implementability constraint, as reflected in the two components of the "incentive term."

As it turns out, at the optimum $\hat{\xi}_t$ becomes zero. Intuitively, once the level of reserves is large enough, the borrowing constraint becomes binding, and thus households cannot offset the government policy. When optimizing, the government fine-tunes the accumulation of reserves so that the borrowing constraint becomes binding exactly at the level of tradable consumption that corresponds to the constrained-efficient allocation. This result is a corollary of the proposition below.

Proposition C2. *Suppose Assumption 1 holds. Then, the solution to the optimal reserve accumulation policy presented in Problem 1 achieves the same utility as the constrained-efficient allocations. Moreover, the optimal policy is time consistent.*

Proof. We guess and verify that (C.5)-(C.7) are slack, and so $\hat{\xi}_t = \hat{\chi}_t = \hat{\nu}_t = 0$. Using this conjecture,

and combining (C.8) and (C.10), we arrive at

$$u_T(c_t^T, y^N) = \beta R_t \mathbb{E}_t[u_T(c_{t+1}^T, y^N) + \hat{\mu}_{t+1}\Psi_{t+1}] + \hat{\mu}_t(1 - \Psi_t). \tag{C.15}$$

By Assumption 1, $\Psi_t < 1$ and since $\hat{\mu}_t \geq 0$, $\hat{\mu}_{t+1} \geq 0$, and $\Psi_{t+1} \geq 0$, we have that:

$$u_T(c_t^T, y^N) - \beta R_t \mathbb{E}_t u_T(c_{t+1}^T, y^N) = \beta R_t \mathbb{E}_t\left[\hat{\mu}_{t+1}\Psi_{t+1}\right] + \hat{\mu}_t(1 - \Psi_t) \geq 0. \tag{C.16}$$

Setting $\mu_t = u_T(c_t^T, y^N) - \beta R_t \mathbb{E}_t u_T(c_{t+1}^T, y^N)$ and using (C.16), it follows that $\mu_t \geq 0$, so (C.7) is satisfied. Moreover, from (C.12), it follows that (C.6) is satisfied as well as conjectured.

Finally, (C.8) and (C.9) imply that (C.4) binds if and only if (C.3) binds, so that (C.3) and (C.4) can be combined to deliver

$$\frac{b_{t+1} - A_{t+1}}{R_t} \leq \kappa_t \left(y_t^T + \frac{1-\omega}{\omega}\left(\frac{c_t^T}{y^N}\right)^{\eta+1} y^N \right). \tag{C.17}$$

Using $b_{t+1} - A_{t+1} = b_{t+1}^\star$, we can see that (C.17) is equivalent to the borrowing constraint in the constrained-efficient problem (13). Therefore, Problem 1 reduces to the same constrained-efficient planning problem of Section 3.4. Hence, it follows that the optimal reserve policy achieves the constrained-efficient allocations and is time consistent. ∎

D Model Extensions

In this section, we present different model extensions discussed in the main text.

D.1 Separate fiscal and monetary authorities

For future reference, note that in addition to optimality conditions (5)-(7), the household's optimal plan now also has to satisfy the Euler equation for government bonds

$$\lambda_t = \beta R_t^g \mathbb{E}_t \lambda_{t+1} + \mu_t^g \quad \text{with } \mu_t^g = 0 \text{ if } b_{t+1}^g > 0\,, \tag{D.1}$$

where μ_t^g denotes the household's multiplier on its borrowing constraint for government bonds.

Proposition D3. *Let A^{max} denote the maximum level of reserves ever required by equation (RP) along the equilibrium path, given initial net foreign liabilities $b_0 - A_0$. Further, let $W_t \equiv A_t + B_t^g$ denote the beginning of period wealth of the central bank. Then, if $\bar{B}^g > W_0 > A^{max}$, the central bank*

policy $A_{t+1} = A^{\star}_{t+1}$ *and*

$$T^{CB}_t = W_t \left(1 - \frac{1}{R^g_t}\right) - A^{\star}_{t+1}\left(\frac{1}{R_t} - \frac{1}{R^g_t}\right) \tag{D.2}$$

is feasible (i.e., satisfies $T^{CB}_t \geq 0 \; \forall t$*) and achieves the constrained-efficient allocation.*

Proof. The first step is to show that under the proposed policy, the central bank's beginning of period wealth is constant over time at $W_t = W_0$. Substituting $A_{t+1} = A^{\star}_{t+1}$ and (D.2) into the central bank's budget constraint (25) yields

$$\frac{A_{t+1}}{R_t} + \frac{B^g_{t+1}}{R^g_t} + W_t\left(1 - \frac{1}{R^g_t}\right) - A_{t+1}\left(\frac{1}{R_t} - \frac{1}{R^g_t}\right) = W_t.$$

Rearranging yields $W_{t+1} \equiv A_{t+1} + B^g_{t+1} = W_t$, from which it follows that $W_t = W_0 \; \forall t$.

Since $B^g_{t+1} = W_t - A_{t+1}$ and $A_{t+1} \in [0, A^{max}]$, it must be that $B^g_{t+1} \in (0, \bar{B}^g)$. From the market clearing condition for government bonds, it must therefore be that $b^g_{t+1} \in (0, \bar{B}^g)$, so the household's Euler equation for government bonds (D.1) must hold with equality:

$$R^g_t = \frac{u_T(t)}{\beta\, \mathbb{E}_t u_T(t+1)}. \tag{D.3}$$

Using $A_{t+1} = A^{\star}_{t+1}$ and consolidating the central bank's and fiscal authority's budget constraints, we obtain

$$\frac{A^{\star}_{t+1}}{R_t} - A^{\star}_t = \frac{\bar{B}^g - B^g_{t+1}}{R^g_t} - (\bar{B}^g - B^g_t) + T_t \equiv \mathcal{T}_t.$$

Substituting the above expression into the household's budget constraint, it is apparent that conditional on the household's equilibrium government bond holding choice, the household problem is equivalent to that of our baseline model, where T_t is replaced by $\mathcal{T}_t$. As a result, Proposition 1 applies and the constrained efficient allocation is achieved.

It remains to show that the central bank policy is feasible, that is $T^{CB}_t \geq 0$ at all times. Adding and subtracting $A^{\star}_{t+1}$ on the right-hand side of (D.2) yields

$$T^{CB}_t = \left(W_t - A^{\star}_{t+1}\right)\left(1 - \frac{1}{R^g_t}\right) + A^{\star}_{t+1}\left(1 - \frac{1}{R_t}\right).$$

The first term on the right-hand side is positive since $A^{\star}_{t+1} \leq A^{max} < W_0 = W_t$ and $R^g_t \geq R_t > 1$. The second term is non-negative since $A^{\star}_{t+1} \geq 0$ and $R_t > 1$. It follows that T^{CB}_t is necessarily positive and the central bank policy is feasible. ∎

D.2 Distortionary taxes

We consider a modified problem of the government that chooses a state-contingent sequence $\{A_{t+1}\}_{t=0}^{\infty}$ to maximize welfare in the competitive equilibrium subject to a cost of lump sum taxes or transfers. The government's budget constraint is still

$$T_t = \frac{A_{t+1}}{R_t} - A_t. \tag{D.4}$$

However, the tradable resource constraint is now

$$c_t^T \leq y^T - T_t - \Phi(T_t) + \frac{b_{t+1}}{R_t} - b_t, \tag{D.5}$$

where the convex function $\Phi(\cdot)$ captures the distortionary cost of taxes or transfers. Using again $U(c^T, c^N)$ to denote $u(c(c^T, c^N))$, the problem consists of solving:

Problem 2 (Optimal Policy under distortionary taxes)**.**

$$\max_{b_{t+1}, A_{t+1}, c_t^T, \mu_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t^T, y^N) \tag{D.6}$$

subject to

$$c_t^T = y^T + A_t - \frac{A_{t+1}}{R_t} - \Phi\left(\frac{A_{t+1}}{R_t} - A_t\right) + \frac{b_{t+1}}{R_t} - b_t, \qquad (\hat{\lambda}_t) \tag{D.7}$$

$$\frac{b_{t+1}}{R_t} \leq \kappa \left(y_t^T + \frac{1-\omega}{\omega} \left(\frac{c_t^T}{y^N} \right)^{\eta+1} y^N \right) \qquad (\hat{\mu}_t) \tag{D.8}$$

$$A_{t+1} \geq 0, \qquad (\hat{\zeta}_t) \tag{D.9}$$

$$u_T(c_t^T, y^N) = \beta R_t \mathbb{E}_t u_T(c_{t+1}^T, y^N) + \mu_t, \qquad (\hat{\xi}_t) \tag{D.10}$$

$$0 = \mu_t \left[\frac{b_{t+1}}{R_t} - \kappa \left(y^T + \frac{1-\omega}{\omega} \left(\frac{c_t^T}{y^N} \right)^{\eta+1} y^N \right) \right], \qquad (\hat{\chi}_t) \tag{D.11}$$

$$\mu_t \geq 0. \qquad (\hat{\nu}_t) \tag{D.12}$$

where $\hat{\lambda}_t, \hat{\mu}_t, \hat{\xi}_t, \hat{\zeta}_t, \hat{\chi}_t$ and $\hat{\nu}_t$ are the multipliers associated with the constraints (D.7)-(D.12).

The government's first-order conditions for b_{t+1}, A_{t+1}, c_t^T and μ_t are given by

$$b_{t+1}: \quad \frac{\hat{\lambda}_t}{R_t} + \frac{\hat{\chi}_t \mu_t}{R_t} = \beta \mathbb{E}_t \hat{\lambda}_{t+1} + \frac{\hat{\mu}_t}{R_t}, \tag{D.13}$$

$$A_{t+1}: \quad \frac{\hat{\lambda}_t}{R_t}\left[1 + \Phi'\left(\frac{A_{t+1}}{R_t} - A_t\right)\right] = \beta \mathbb{E}_t \left[\hat{\lambda}_{t+1}\left(1 + \Phi'\left(\frac{A_{t+2}}{R_{t+1}} - A_{t+1}\right)\right)\right] + \hat{\zeta}_t, \tag{D.14}$$

$$c_t^T: \quad \hat{\lambda}_t = u_T(t) + \hat{\xi}_t u_{TT}(c_t^T, y^N) - \hat{\xi}_{t-1} R_{t-1} u_{TT}(c_t^T, y^N) + \Psi_t(\hat{\mu}_t - \hat{\chi}_t \mu_t), \tag{D.15}$$

$$\mu_t: \quad \hat{\xi}_t = \hat{\chi}_t \left[\frac{b_{t+1}}{R_t} - \kappa\left(y^T + \frac{1-\omega}{\omega}\left(\frac{c_t^T}{y^N}\right)^{\eta+1} y^N\right)\right] + \hat{\nu}_t, \tag{D.16}$$

and complementary slackness conditions

$$0 = \hat{\mu}_t \left[\frac{b_{t+1}}{R_t} - \kappa\left(y^T + \frac{1-\omega}{\omega}\left(\frac{c_t^T}{y^N}\right)^{\eta+1} y^N\right)\right], \tag{D.17}$$

$$0 = \hat{\zeta}_t A_{t+1}. \tag{D.18}$$

We can again combine conditions (D.14) and (D.15) to compute the planner's Euler equation. In a state in which (D.11)-(D.12) are not binding, and assuming that the constraints for reserves and borrowing are currently slack, we now have that

$$u_T(t) = \frac{\beta R_t}{1 + \Phi'\left(\frac{A_{t+1}}{R_t} - A_t\right)} \mathbb{E}_t \left[\left(1 + \Phi'\left(\frac{A_{t+2}}{R_{t+1}} - A_{t+1}\right)\right)\left(u_T(t+1) + \hat{\mu}_{t+1}\Psi_{t+1} + \hat{\xi}_{t+1} u_{TT}(c_{t+1}^T, y^N) - \hat{\xi}_t R_t u_{TT}(c_{t+1}^T, y^N)\right)\right]$$
$$+ \hat{\xi}_{t-1} R_{t-1} u_{TT}(c_t^T, y^N) - \hat{\xi}_t u_{TT}(c_t^T, y^N). \tag{D.19}$$

While the optimality conditions are similar to those discussed in Appendix C, distortionary taxation introduces two wedges in the planner's Euler equation for reserves relative to the baseline model related to the marginal cost of raising tax revenues.

We solve a version of the model at the calibrated parameters and assume an asymmetric quadratic cost function:

$$\Phi\left(\frac{A_{t+1}}{R_t} - A_t\right) = \phi \times \left(\frac{A_{t+1}}{R_t} - A_t\right)^2 \mathbb{1}_{\frac{A_{t+1}}{R_t} > A_t}$$

This specification assumes quadratic costs when the government must tax the household to increase reserves but zero costs when the reserve adjustment leads to positive transfers to the households. We consider different values for the parameter ϕ and compute the optimal level

of reserve accumulation, as well as the average cost paid at the ergodic distribution and the curvature of the cost. The results are presented in Table D.1. The first column shows that average reserve accumulation is between 50 and 70 percent of the optimal value in an economy with no distortionary costs. The second column shows that the average costs paid are small for all values of ϕ. The third column shows that the costs are nonetheless strongly convex: the marginal cost of taxation rises quickly with the size of the reserve purchase.

Table D.1: Reserve accumulation under distortionary taxation

ϕ	Reserve Accumulation % of baseline	Average Cost % of interest earned	Cost Curvature % of interest earned
0.05%	70.90	0.15	2.55
0.1%	70.33	0.23	5.10
1%	55.06	1.45	51.01
2%	52.07	2.40	102.01

Note: Reserve accumulation is average reserves relative to the economy without distortionary costs. With $T_t = qA_{t+1} - A_t$ and $q = e^{-\bar{r}}$, average cost is $\Phi(T_t)/T_t = \phi T_t$ relative to the interest earned per unit, $1 - q$, averaged over periods with $T_t > 0$ at the ergodic distribution. Cost curvature is $\Phi''(T_t)/(1-q) = 2\phi/(1-q)$: the increase in the marginal cost of taxation, relative to the interest earned, per unit increase in T_t.

D.3 Production

We assume that households are endowed with a fixed amount of hours $\bar{h}$, and do not value leisure. They receive a competitive wage w_t for their labor, as well as profits from firms in the tradable and nontradable sectors, π_t^T and π_t^N. The household's budget and credit constraint are given by:

$$c_t^T + p_t^N c_t^N - \frac{b_{t+1}}{R_t} = w_t \bar{h} + \pi_t^T + \pi_t^N - b_t - T_t, \tag{D.20}$$

$$\frac{b_{t+1}}{R_t} \leq \kappa_t \left(w_t \bar{h} + \pi_t^T + \pi_t^N \right). \tag{D.21}$$

The tradable and nontradable goods are produced by competitive firms that maximize profits and solve:

$$\max_{h_t^T} z_t^T (h_t^T)^\alpha - w_t h_t^T \tag{D.22}$$

$$\max_{h_t^N} p_t^N z^N (h_t^N)^\alpha - (1 + \tau_t^N) w_t h_t^N + T_t^N, \tag{D.23}$$

where z_t^T is a stochastic productivity shock, z^N and α are constant parameters, and τ_t^N is a labor tax in the non-tradable sector, to be rebated lump-sum via a transfer T_t^N to non-tradable goods-producing firms.

The households' optimality conditions are still given by (5), (6) and (7). The tradable and non-tradable goods firms' optimality conditions are respectively given by

$$w_t = z_t^T \alpha \left(h_t^T\right)^{\alpha-1}, \tag{D.24}$$

$$(1+\tau_t^N) w_t = p_t^N z^N \alpha \left(h_t^N\right)^{\alpha-1}. \tag{D.25}$$

The competitive equilibrium is given by sequences of consumption, labor, wages and prices of non-tradables such that all optimality conditions are satisfied and market clearing holds for all goods and labor:

$$c_t^T - \frac{b_{t+1}}{R_t} = z_t^T (h_t^T)^\alpha - b_t, \tag{D.26}$$

$$c_t^N = z^N (h_t^N)^\alpha, \tag{D.27}$$

$$\bar{h} = h_t^N + h_t^T. \tag{D.28}$$

The planner's problem, assuming that the planner controls the labor allocation and borrowing (see Arce et al., 2023), is:

$$V(b, z^T, R, \kappa) = \max_{c^T, c^N, h^T, h^N, b'} u(c^T, c^N) + \beta \mathbb{E} V(b', z^{T\prime}, R', \kappa') \tag{D.29}$$

subject to

$$c^T - \frac{b'}{R} = z^T \left(h^T\right)^\alpha - b,$$

$$c^N = z^N \left(h^N\right)^\alpha,$$

$$\bar{h} = h^T + h^N,$$

$$\frac{b'}{R} \leq \kappa \left[z^T \left(h^T\right)^\alpha + \frac{1-\omega}{\omega} \left(\frac{c^T}{c^N}\right)^{\eta+1} z^N \left(h^N\right)^\alpha \right].$$

Table 5 presents the results for the reserve accumulation that implements the planner's problem.

D.4 Unsecured Borrowing

In this appendix, we extend our baseline model by giving households access to an additional, unsecured bond, $\tilde{b}$, with a generic interest rate schedule $\tilde{R}(\tilde{B}_{t+1})$.[46] This extension shows that even when households do not face a hard borrowing constraint, reserve accumulation remains effective at improving the economy's net foreign asset position and reducing its vulnerability to financial crises.

The budget constraint of the household is now given by

$$c_t^T + p_t^N c_t^N - \frac{b_{t+1}}{R} - \frac{\tilde{b}_{t+1}}{\tilde{R}(\tilde{B}_{t+1})} = y_t^T + p_t^N y^N - b_t - \tilde{b}_t - T_t, \tag{D.30}$$

where b continues to refer to secured borrowing subject to a collateral constraint, and $\tilde{b}$ refers to the additional unsecured borrowing. For simplicity, we assume that R is constant.

The household's optimal choice for $\tilde{b}$ implies the Euler equation

$$u_T(t) = \beta \tilde{R}_t \mathbb{E}_t u_T(t+1). \tag{D.31}$$

Assume that $\tilde{R}$ is continuously differentiable with $\tilde{R}(\tilde{B}_{t+1}) = R$ if $\tilde{B}_{t+1} \leq 0$ and $\tilde{R}'(\tilde{B}_{t+1}) > 0$ for $\tilde{B}_{t+1} > 0$. That is, as households borrow, the interest rate rises above the risk-free rate. It then follows from (D.31) and (7) that households choose to borrow $\tilde{b}_{t+1} > 0$ if and only if they are borrowing constrained.

In our baseline model, when the central bank accumulates reserves, this drives households to the borrowing constraint, which prevents them from borrowing more. Now, as a result of reserve accumulation, households would increase their borrowing as reflected by (D.31). The cost from this policy is that the interest rate at which the SOE borrows increases.[47] However, to the extent that $\tilde{R} > R$, households raise borrowing less than one to one with reserves. As a result, reserves remain effective in this extended setup.

To see that reserves are also desirable, consider the planner's problem in this environment:

$$V(b, \tilde{b}, A, y^T, \kappa) = \max_{b', \tilde{b}', A', c^T} u(c(c^T, y^N)) + \beta \mathbb{E} V(b', \tilde{b}', A', y^{T\prime}, \kappa') \tag{D.32}$$

subject to

[46] We assume that the interest rate schedule is a function of aggregate unsecured borrowing, but our results extend to the case where it is a function of individual variables. Possible ways to endogenize this schedule stem from balance sheet constraints on intermediaries (Gabaix and Maggiori, 2015) or portfolio costs (Bianchi and Lorenzoni, 2021).

[47] See also Amador et al. (2018) and Fanelli and Straub (2020).

$$b + \tilde{b} + c^T = y^T + A + \frac{b' - A'}{R} + \frac{\tilde{b}'}{\tilde{R}(\tilde{b}')}, \tag{D.33}$$

$$\frac{b'}{R} \leq \kappa \left[y^T + \frac{1-\omega}{\omega} \left(\frac{c^T}{y^N} \right)^{\eta+1} y^N \right], \tag{D.34}$$

$$u_T(c^T, y^N) = \beta \tilde{R}(\tilde{b}') \mathbb{E} u_T \left(c^T(b', \tilde{b}', A', y^{T\prime}, \kappa'), y^N \right), \tag{D.35}$$

where (D.33) reflects the resource constraint for tradables, which follows from combining (D.30) and (8), and (D.35) reflects the implementability constraint associated with households' choice of $\tilde{b}$.

Consider a state where (D.34) is not binding. If the planner sets $\tilde{b}' = 0$, this implies from (D.35) that:

$$u_T(c^T, y^N) = \beta R \mathbb{E} u_T(c^T_{t+1}, y^N) \tag{D.36}$$

which implies that there is no active government intervention to raise the NFA position through reserve accumulation.

However, if the borrowing constraint binds with positive probability next period, (21) implies that

$$u_T(c^T, y^N) > \beta R \mathbb{E} u_T(c^T_{t+1}, y^N). \tag{D.37}$$

Comparing (D.36) and (D.37), we reach a contradiction. That is, it must be optimal to induce $\tilde{b}' > 0$ and thus $A' > 0$.

Intuitively, when households raise their unsecured borrowing starting from $\tilde{b} = 0$, the fact that they do so at a rate $\tilde{R} > R$ induces a second-order welfare loss. To the extent that there is a first-order gain from raising the NFA because of the pecuniary externality, the planner finds it optimal to accumulate reserves. These results highlight that the desirability of foreign reserves in our baseline model extends more generally to environments where households do not face a hard borrowing constraint.

E Verifying Uniqueness of Equilibrium

Open economy models with a collateral constraint like the one we study may feature multiple equilibria, as formally established by Schmitt-Grohé and Uribe (2021). They provide necessary and sufficient conditions under which a non-stochastic version of the model will feature equilibrium multiplicity. Our parameterization does not fall within the conditions that would allow us to determine unambiguously that there is a unique equilibrium or there are multiple equilibria.

Because of this, we provide a numerical algorithm designed to check whether our equilibrium is unique. For our calibration, we find that there are no other equilibria.

We begin from the solution to the competitive equilibrium absent any intervention, which we solve using time iteration, following Bianchi (2011).[48] We denote the equilibrium law of motion for debt as $\mathcal{B}(b,s)$. We construct a grid for possible values of debt $B_N = [b_1....b_{max}]$. For the upper bound of the grid, we take a value arbitrarily close to the natural debt limit. The natural debt limit can be obtained as the fixed point of the following problem:

$$\begin{aligned} b_{\max}(s) &= \max_{b'} y^T + \frac{b'}{R} \\ &\text{subject to} \\ \frac{b'}{R} &\leq \kappa \left[y^T + \frac{1-\omega}{\omega} \left(\frac{y^T + \frac{b'}{R} - b_{\max}}{y^N} \right)^{1+\eta} y^N \right] \end{aligned} \tag{E.1}$$

Considering that this condition must be satisfied for every possible shock, we obtain $b_{max} = y_{min}(1+\kappa)$.

For every point in the grid for initial debt and shocks, we then check whether the following conditions are satisfied for every $b' \in B_N$ other than $\mathcal{B}(b,s)$.

$$u_T(y^T - b + b'/R_t, y^N) = \beta R_t E u_T(c_T(b',s'), y^N) + \mu \tag{E.2}$$

$$\frac{b'}{R_t} \leq \kappa \left[\frac{1-\omega}{\omega} \left(\frac{y^T - b + b'/R_t}{y^N} \right)^{\eta+1} y^N + y^T \right] \tag{E.3}$$

$$\mu \geq 0, \qquad \mu \left(\kappa \left[\frac{1-\omega}{\omega} \left(\frac{y^T - b + b'/R_t}{y^N} \right)^{\eta+1} y^N + y^T \right] - \frac{b'}{R_t} \right) = 0 \tag{E.4}$$

If there is another $b' \in B_N$ that satisfies these conditions, then there are multiple equilibria.

Following this procedure, we do not find any state with more than one solution to the system of equations. To illustrate this result, we turn to Figure E.1. In the two panels of the figure, the red line computes the excess borrowing capacity, that is the difference between the debt limit and the issued level of debt (E.3), for each value on the grid. The blue line uses (E.2) to calculate the Lagrange multiplier associated with the collateral constraint (μ) consistent with each potential debt level. In an equilibrium with a binding collateral constraint, the excess borrowing capacity is zero

[48] We use 100 equally spaced points in the grid for bonds between 0 and the natural debt limit, 20 values for the tradable endowment shock, and 17 values for the financial shock. We interpolate linearly for values of next-period bonds not in the grid. To solve the constrained efficient economy, we use Lagrange multiplier iteration. Specifically, we compute the planner's λ_t to solve for the optimal policies and update our policies using equation (15).

and the Lagrange multiplier associated with the collateral constraint (μ) is positive. Conversely, in an equilibrium with a non-binding collateral constraint, the excess borrowing capacity is strictly positive and the Lagrange multiplier associated with the collateral constraint is zero.

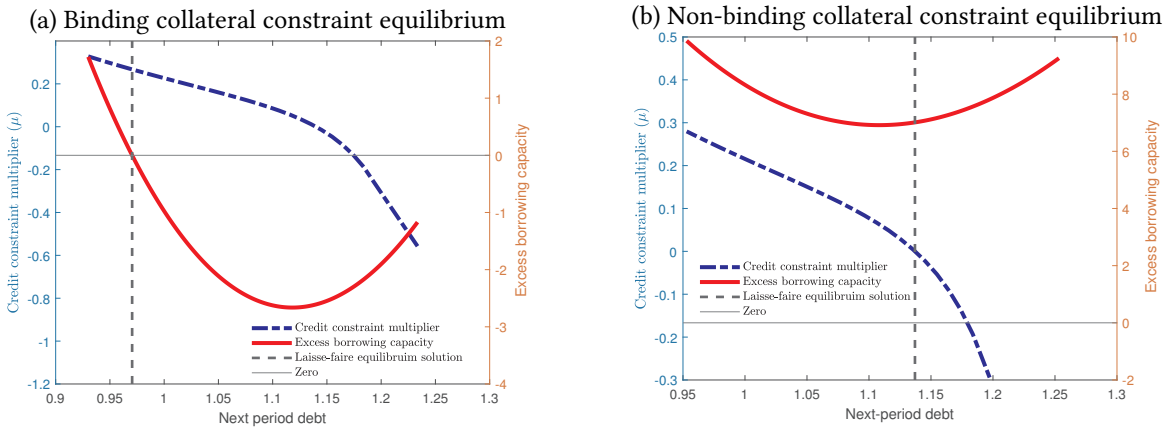


Figure E.1: Verifying Uniqueness

Note: Solutions to the optimality conditions for the states with the highest values of the financial shock where the collateral constraint binds and an endowment of tradables two standard deviations below its mean

Panel A plots the solutions to equations (E.2) and (E.3) for the state with an endowment of tradables two standard deviations below its mean and the highest value of the parameter κ such that the collateral constraint still binds for high enough levels of initial debt. As we can see in panel A, a higher level of next period debt than the one that solves the competitive equilibrium may also satisfy (E.3) with equality (zero excess borrowing capacity). However, at those higher values of debt, the Lagrange multiplier associated with the constraint is not positive. Similarly, a lower level of end of period debt will be consistent with a positive excess borrowing capacity, but at those values, the Lagrange multiplier μ is strictly positive.

Panel B plots instead an equilibrium at the same exogenous state as panel A but where the initial level of debt is smaller and the competitive equilibrium no longer exhibits a binding collateral constraint. As one can see, it is not possible to find an equilibrium with binding collateral constraints, since excess borrowing capacity is never zero in this case.

So we have established that under our parameterization, we can only find one equilibrium. The only caveat of this procedure is that we take as given a continuation equilibrium from $t > 0$. However, given a continuation equilibrium for $t > 0$, the procedure can exhaustively determine whether there is multiplicity or not at $t = 0$ for every point in the grid.

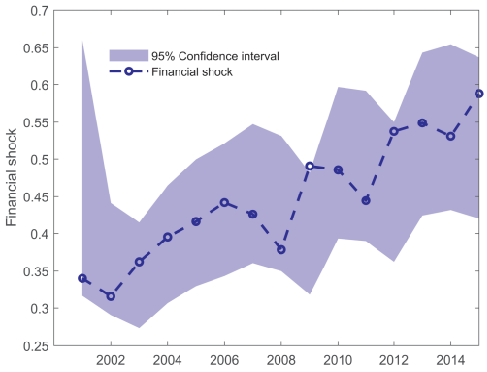
F Financial shocks

In the dynamic exercise presented in Figure 8 we feed to the regulated economy a sequence of financial shocks (κ_t) that is consistent with our assumption that Mexico was following optimal reserve accumulation policies during those years. As explained in Section 4.1, the financial shock follows a first-order autoregressive process of the form:

$$\log(\kappa_t) = (1 - \rho^{\kappa}) \log(\bar{\kappa}) + \rho^{\kappa} \log(\kappa_{t-1}) + \varepsilon_t^{\kappa}$$

Figure F.1 plots this sequence of kappa shocks (panel a) as well as the innovations of this autoregressive process (panel b). We also plot the 95% confidence intervals associated with each observation.

(a) Financial shock



(b) Innovations in the financial shock

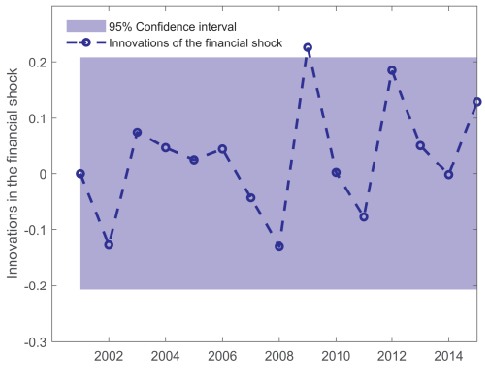


Figure F.1: Financial shock and innovations of the financial shock in the dynamic exercise

Note: For the first observation of the financial shock, the bounds of the 95% confidence intervals are computed using the unconditional mean and standard deviation of the auto-regressive process.

G Reserve policy with a general financial friction

In this appendix, we replace the collateral constraint of the main text with a general price schedule for private debt, and provide conditions under which the government does or does not find it optimal to accumulate reserves.

Households borrow by issuing one-period bonds. A bond issued at date t promises one unit of tradables at date $t+1$ and sells at the price

$$q_t = q\big(b_{t+1},\, B_{t+1},\, A_{t+1};\; B_t,\, A_t\big), \tag{G.1}$$

where b_{t+1} is the household's own debt due at date $t+1$, B_{t+1} is aggregate private debt due at date $t+1$, A_{t+1} is the stock of reserves that the government carries into date $t+1$, and B_t and A_t are the stocks of private debt and reserves due at date t. The schedule therefore allows the cost of borrowing to depend both on the positions chosen in the current period and on the positions inherited from the previous one. We take q to be twice continuously differentiable, and we assume that $q_t \leq 1/R$, where R is the world gross interest rate.[49] We also assume that the resources raised by borrowing, $q_t b_{t+1}$, are weakly concave in the household's own debt b_{t+1}.

Households choose $\{c_t^T, c_t^N, b_{t+1}\}$ to maximize (2) subject to their budget constraint, taking as given prices, taxes, and the aggregate arguments of the schedule. Their optimality conditions are the static condition (5) and the Euler equation

$$\left[q_t + \frac{\partial q_t}{\partial b_{t+1}}\, b_{t+1}\right] u_T(t) = \beta\, \mathbb{E}_t\, u_T(t+1), \tag{G.2}$$

where the bracket is the amount of resources raised by promising one more unit of repayment: the price of the marginal unit plus the change in the price of all the units issued.

As in our baseline model, this economy features a pecuniary externality. Households take aggregate debt as given and therefore do not internalize that their borrowing affects the price faced by all other households, in the current period through $\partial q_t/\partial B_{t+1}$ and in the next one through $\partial q_{t+1}/\partial B_{t+1}$.[50]

[49] One possible microfoundation is one in which international lenders face financial frictions, as in Gabaix and Maggiori (2015).

[50] In our baseline model, the market price of bonds is fixed at $1/R$, but households' decisions affect the shadow cost of borrowing by other households through the collateral constraint. While the baseline is not itself a member of the class (G.1), since the price is not differentiable at the collateral limit, it can be approached by smooth schedules in which the resources raised by borrowing converge to a kink at the collateral value, and the analysis below applies to it in that limit.

G.1 Optimal reserve policy

We now consider the problem of a government that chooses reserves to maximize household welfare in the competitive equilibrium. Following the approach of Appendix C, we write this problem as that of a planner who chooses $\{c_t^T, B_{t+1}, A_{t+1}\}_{t\geq 0}$ subject to the economy's resource constraint for tradables, the government's budget constraint, and market clearing for non-tradables, the households' Euler equation (G.2), which enters as an implementability constraint, and the non-negativity of reserves:

$$\max_{\{c_t^T, B_{t+1}, A_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \, u\big(c(c_t^T, y^N)\big)$$

subject to

$$c_t^T + B_t + \frac{A_{t+1}}{R} \leq y_t^T + A_t + q_t \, B_{t+1}, \tag{G.3}$$

$$u_T(t)\left[q_t + \frac{\partial q_t}{\partial b_{t+1}} B_{t+1}\right] = \beta \, \mathbb{E}_t \, u_T(t+1), \tag{G.4}$$

$$A_{t+1} \geq 0. \tag{G.5}$$

Here and below, the schedule and its derivatives are evaluated at $b_{t+1} = B_{t+1}$. Note that the positions chosen at date t affect not only the current price q_t but also next period's price q_{t+1}, in which B_{t+1} and A_{t+1} enter as the inherited stocks.

Let $\beta^t \lambda_t$, $\beta^t \nu_t$, and $\beta^t \zeta_t$ denote the multipliers on (G.3), (G.4), and (G.5), respectively, with $\lambda_t > 0$ and $\nu_{-1} = 0$. We use primes to denote the choices made at date $t+1$. The first-order conditions with respect to c_t^T, B_{t+1}, and A_{t+1} are

$$c_t^T: \quad u_T(t) - \lambda_t + u_{TT}(t)\left[\nu_t\left(q_t + \frac{\partial q_t}{\partial b_{t+1}} B_{t+1}\right) - \nu_{t-1}\right] = 0, \tag{G.6}$$

$$\begin{aligned} B_{t+1}: \quad & \lambda_t\left[q_t + B_{t+1}\left(\frac{\partial q_t}{\partial b_{t+1}} + \frac{\partial q_t}{\partial B_{t+1}}\right)\right] - \beta \, \mathbb{E}_t \lambda_{t+1} + \Theta_t^B \\ & + \nu_t \, u_T(t)\left[2\frac{\partial q_t}{\partial b_{t+1}} + \frac{\partial q_t}{\partial B_{t+1}} + B_{t+1}\left(\frac{\partial^2 q_t}{\partial b_{t+1}\,\partial b_{t+1}} + \frac{\partial^2 q_t}{\partial b_{t+1}\,\partial B_{t+1}}\right)\right] = 0, \end{aligned} \tag{G.7}$$

$$A_{t+1}: \quad \lambda_t\left[B_{t+1}\frac{\partial q_t}{\partial A_{t+1}} - \frac{1}{R}\right] + \nu_t \, u_T(t)\left[\frac{\partial q_t}{\partial A_{t+1}} + B_{t+1}\frac{\partial^2 q_t}{\partial b_{t+1}\,\partial A_{t+1}}\right] + \beta \, \mathbb{E}_t \lambda_{t+1} + \Theta_t^A = -\zeta_t, \tag{G.8}$$

together with the complementary slackness condition

$$\zeta_t \geq 0, \qquad \zeta_t \, A_{t+1} = 0, \tag{G.9}$$

where

$$\Theta_t^B \equiv \beta\,\mathbb{E}_t\Big[\lambda_{t+1}\,B'\,\frac{\partial q_{t+1}}{\partial B_{t+1}} + \nu_{t+1}\,u_T(t+1)\Big(\frac{\partial q_{t+1}}{\partial B_{t+1}} + B'\,\frac{\partial^2 q_{t+1}}{\partial b'\,\partial B_{t+1}}\Big)\Big], \tag{G.10}$$

$$\Theta_t^A \equiv \beta\,\mathbb{E}_t\Big[\lambda_{t+1}\,B'\,\frac{\partial q_{t+1}}{\partial A_{t+1}} + \nu_{t+1}\,u_T(t+1)\Big(\frac{\partial q_{t+1}}{\partial A_{t+1}} + B'\,\frac{\partial^2 q_{t+1}}{\partial b'\,\partial A_{t+1}}\Big)\Big]. \tag{G.11}$$

By (G.9), the planner accumulates reserves whenever the marginal value of reserves, evaluated at the allocation in which $A_{t+1} = 0$, is positive. This marginal value is the left-hand side of (G.8), and it consists of three terms. The first term, $-\lambda_t/R + \beta\,\mathbb{E}_t\lambda_{t+1}$, is the return on a unit of reserves net of its cost, and it is negative when tradable resources are more valuable in the current period than in the next. It is the counterpart of the multiplier on the collateral constraint of the main text, where $\lambda_t = \beta R\,\mathbb{E}_t\lambda_{t+1} + \mu_t$ with $\mu_t \geq 0$, so that the term equals $-\mu_t/R$. The second term, $\big(\lambda_t B_{t+1} + \nu_t u_T(t)\big)\,\partial q_t/\partial A_{t+1} + \nu_t u_T(t) B_{t+1}\,\partial^2 q_t/\partial b_{t+1}\partial A_{t+1}$, is the effect of reserves on the price of debt issued in the current period. The third term, Θ_t^A, is the effect of reserves on next period's price, which operates through $\partial q_{t+1}/\partial A_{t+1}$.

Rearranging the condition that the marginal value of reserves at $A_{t+1} = 0$ be positive, we obtain a sufficient condition under which reserves are accumulated,

$$\big(\lambda_t B_{t+1} + \nu_t u_T(t)\big)\frac{\partial q_t}{\partial A_{t+1}} + \nu_t u_T(t) B_{t+1}\frac{\partial^2 q_t}{\partial b_{t+1}\,\partial A_{t+1}} + \Theta_t^A \;>\; \frac{\lambda_t}{R} - \beta\,\mathbb{E}_t\lambda_{t+1}. \tag{G.12}$$

Intuitively, the left-hand side collects the sensitivity of borrowing terms to reserves—at issuance through $\partial q_t/\partial A_{t+1}$ and $\partial^2 q_t/\partial b_{t+1}\partial A_{t+1}$, and in the next period through Θ_t^A—while the right-hand side is the cost of a unit of reserves net of its return. At an interior optimum, (G.12) holds with equality.

G.2 When reserve accumulation is undesirable

We present now two examples in which reserve accumulation is undesirable. Suppose that the bond-price schedule takes the form

$$q_t = Q(B_{t+1} - A_{t+1},\, B_t - A_t), \qquad Q_1 < 0, \qquad 0 < Q < 1/R. \tag{G.13}$$

Thus, the price is independent of the household's own debt, but aggregate debt lowers the price through its effect on net liabilities. An equal increase in reserves offsets this price effect, both at

issuance and through the inherited stocks:

$$\frac{\partial q_t}{\partial B_{t+1}} + \frac{\partial q_t}{\partial A_{t+1}} = 0, \qquad \frac{\partial q_{t+1}}{\partial B_{t+1}} + \frac{\partial q_{t+1}}{\partial A_{t+1}} = 0. \tag{G.14}$$

Let $\mathcal{M}_t$ denote the marginal value of reserves, that is, the left-hand side of (G.8). Using the debt first-order condition (G.7) to eliminate $\beta\,\mathbb{E}_t\lambda_{t+1}$ gives

$$\mathcal{M}_t = \lambda_t\Big(q_t - \frac{1}{R}\Big) + \big[\lambda_t B_{t+1} + \nu_t u_T(t)\big]\Big(\frac{\partial q_t}{\partial B_{t+1}} + \frac{\partial q_t}{\partial A_{t+1}}\Big) + \Theta_t^B + \Theta_t^A. \tag{G.15}$$

Since the price is independent of the household's own debt, all second derivatives with respect to b_{t+1} vanish, and the identities in (G.14) imply $\Theta_t^B + \Theta_t^A = 0$ and

$$\mathcal{M}_t = -\lambda_t\Big(\frac{1}{R} - q_t\Big) < 0. \tag{G.16}$$

We then have from (G.8) that $\zeta_t = \lambda_t(1/R - q_t) > 0$, so complementary slackness (G.9) implies $A_{t+1} = 0$ at every date.

The intuition is as follows. A matched one-unit increase in reserves and private debt leaves borrowing terms unchanged because they depend on net debt, but absorbs $1/R - q_t > 0$ units of current resources with no net payoff next period. Reserve accumulation is therefore strictly undesirable, as it incurs a financing cost without correcting the borrowing externality.

In the previous example, it is crucial that reserve accumulation entails a positive financing spread. Reserve accumulation can also be undesirable when the financing spread is zero in normal times, if the associated increase in private borrowing worsens future crisis borrowing terms. Let $s_t \in \{N, C\}$ denote an exogenous Markov state, normal times or crisis, and suppose that

$$q_t = \begin{cases} 1/R, & s_t = N, \\ Q(\gamma B_t - A_t), & s_t = C, \end{cases} \qquad \gamma > 1, \quad Q' < 0, \quad 0 < Q < 1/R.$$

Under this schedule, the government never accumulates reserves in normal times, even though they carry no financing cost.[51] Intuitively, a matched increase in reserves and private debt costs nothing today and has no net payoff next period, but it worsens crisis borrowing terms because the

[51] At a normal-time date, $q_t = 1/R$ does not depend on the positions chosen at t, so (G.7) gives $\beta\,\mathbb{E}_t\lambda_{t+1} - \lambda_t/R = \Theta_t^B$ and the marginal value of reserves in (G.8) is $\mathcal{M}_t = \Theta_t^B + \Theta_t^A$. These positions affect next period's price only in a crisis, where $\partial q_{t+1}/\partial B_{t+1} = \gamma Q'$ and $\partial q_{t+1}/\partial A_{t+1} = -Q'$, so (G.10)–(G.11) yield $\mathcal{M}_t = (\gamma - 1)\,\beta\,\pi_t\,\mathbb{E}_t\big[(\lambda_{t+1}B_{t+2} + \nu_{t+1}u_T(t+1))\,Q'(\gamma B_{t+1} - A_{t+1})\,\big|\,s_{t+1} = C\big]$, where π_t is the probability of a crisis at $t+1$. The term in parentheses is the planner's marginal value of a higher crisis bond price, which we take to be positive (as when $B_{t+2} > 0$ and $\nu_{t+1} \geq 0$). Since $\gamma > 1$ and $Q' < 0$, $\mathcal{M}_t < 0$ whenever $\pi_t > 0$, and (G.9) implies $A_{t+1} = 0$.

additional debt more than offsets the benefit of reserves ($\gamma > 1$). Reserve accumulation therefore backfires.